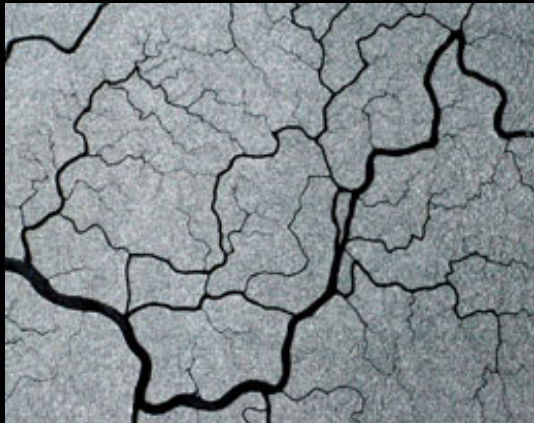
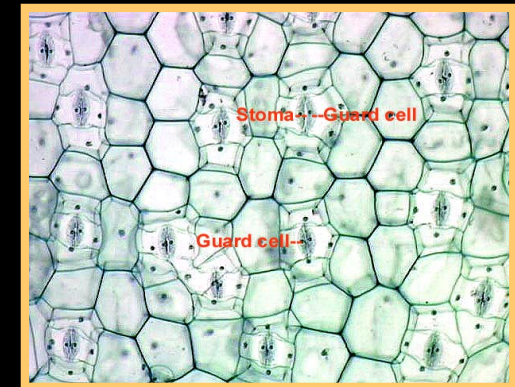


Two classes of real world  
net-like patterns identified  
by angles at junctions



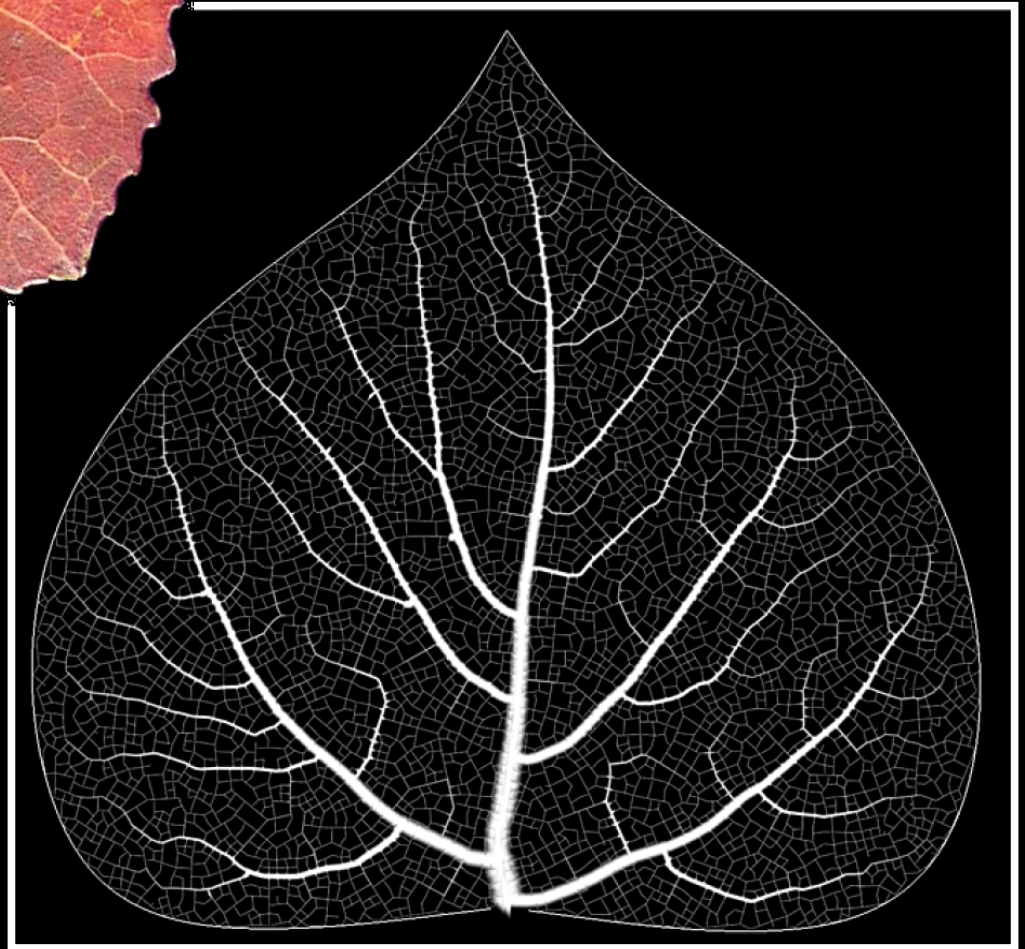
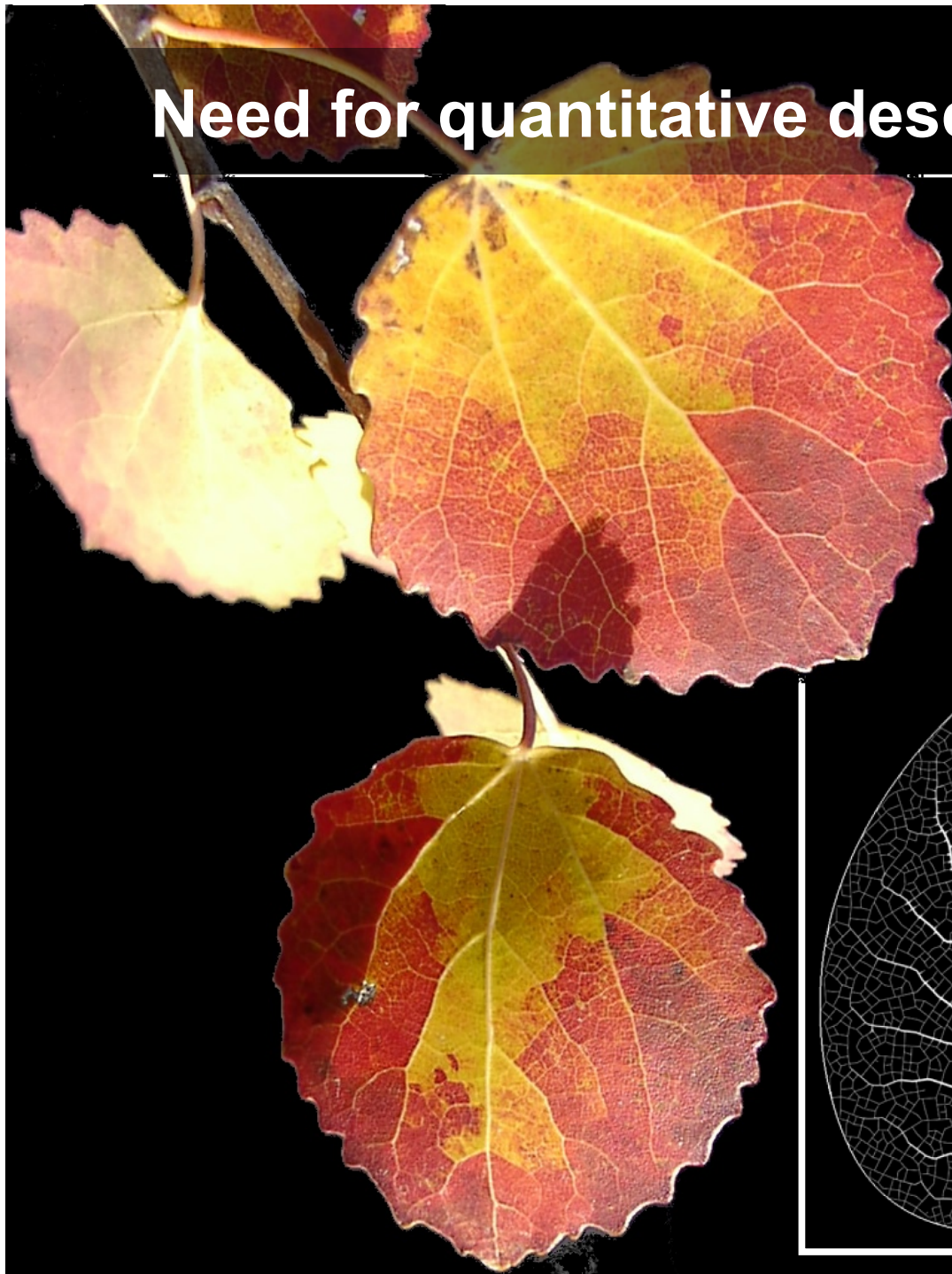
Andrea Perna, Pascale Kuntz, Stéphane Douady





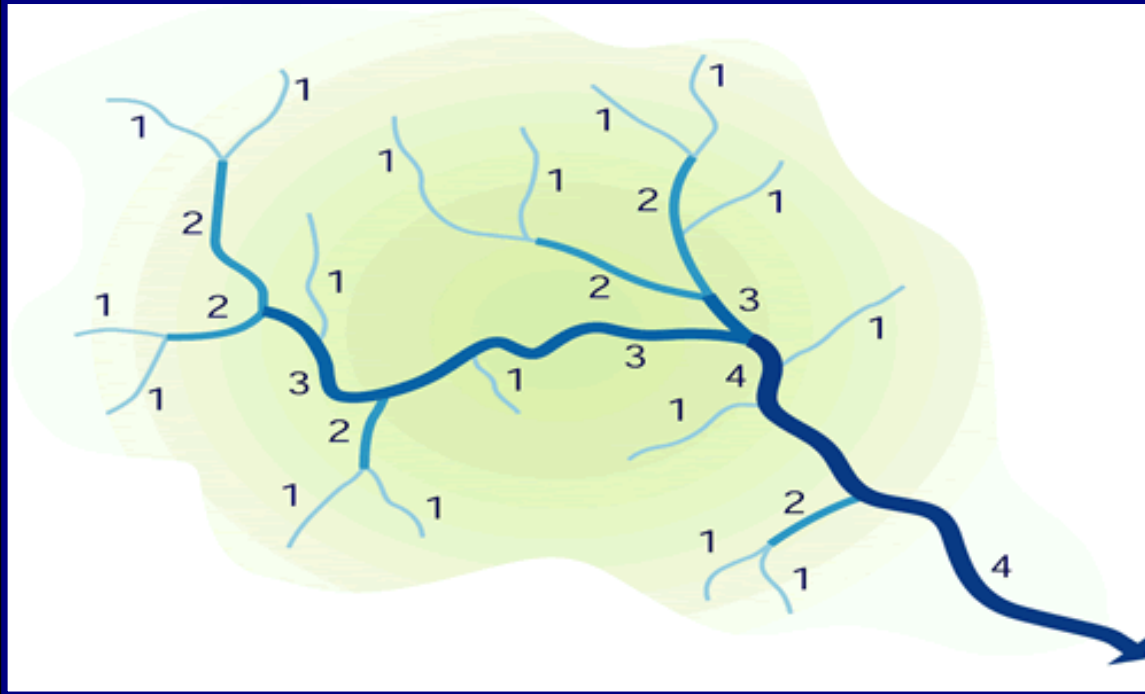
# Need for quantitative description of patterns

---

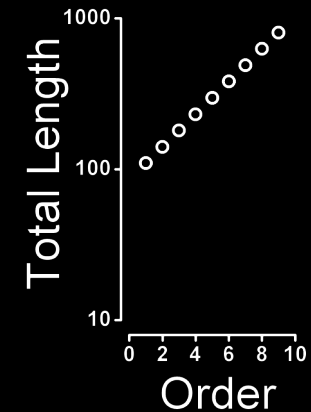
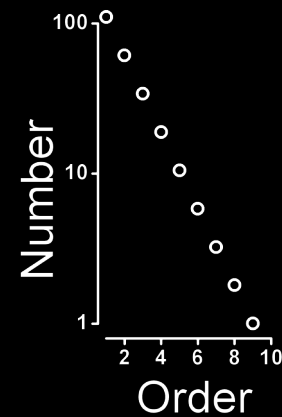
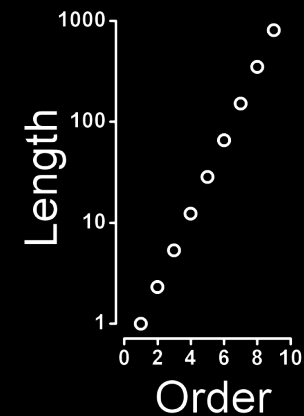




# Difficulty in describing hierarchical patterns

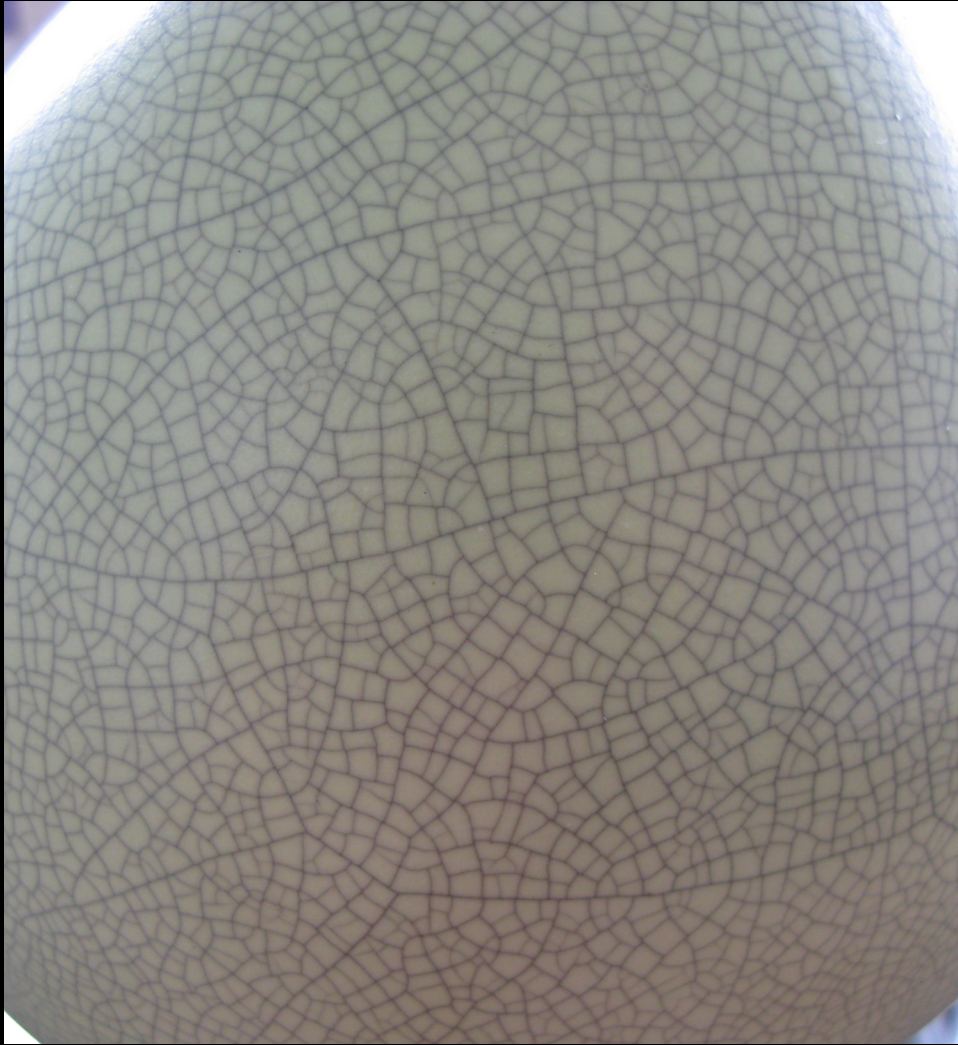


Horton-Strahler classification systems  
for river networks

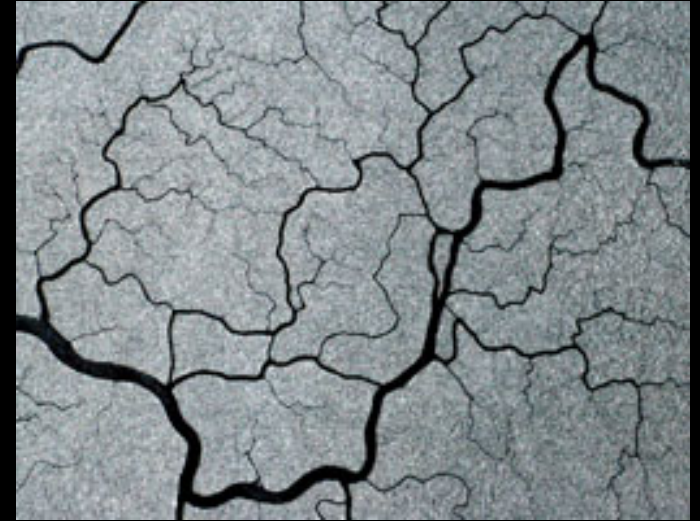




# Difficulty in describing hierarchical patterns



Loops!

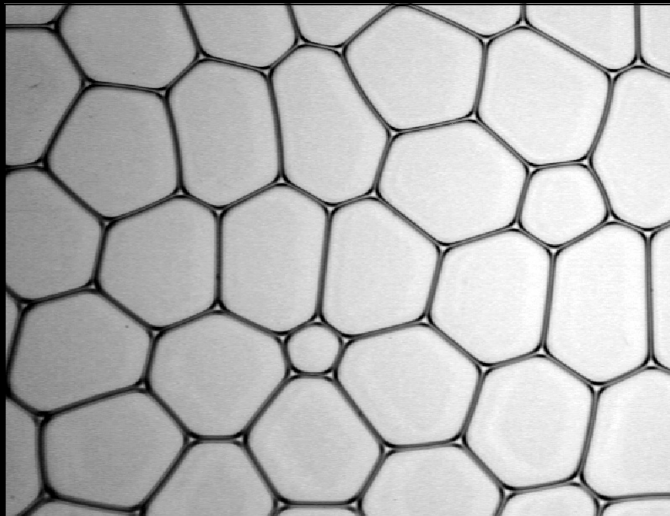
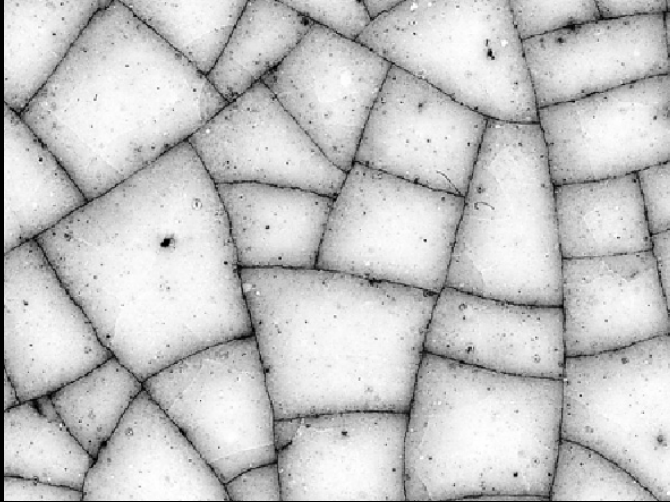


*Mangroves*



## Describe them in terms of graphs

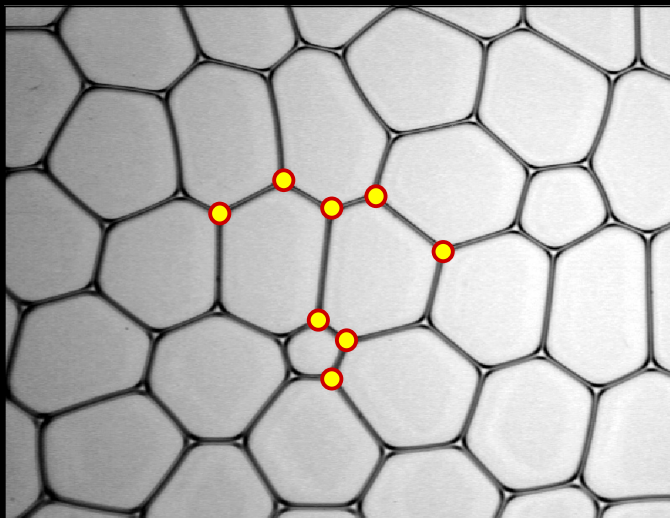
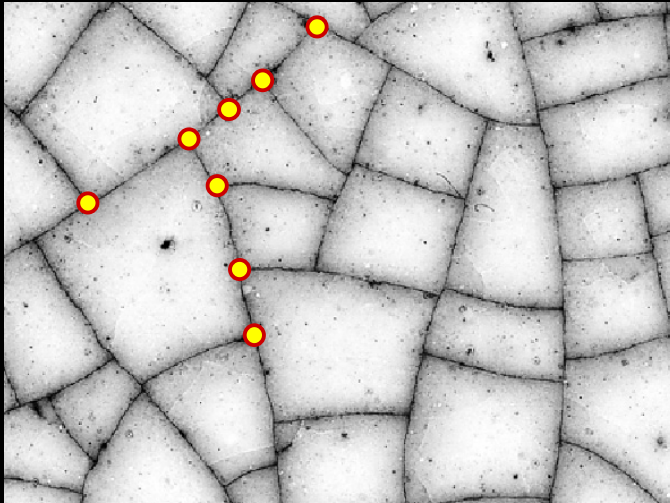
---





## Describe them in terms of graphs

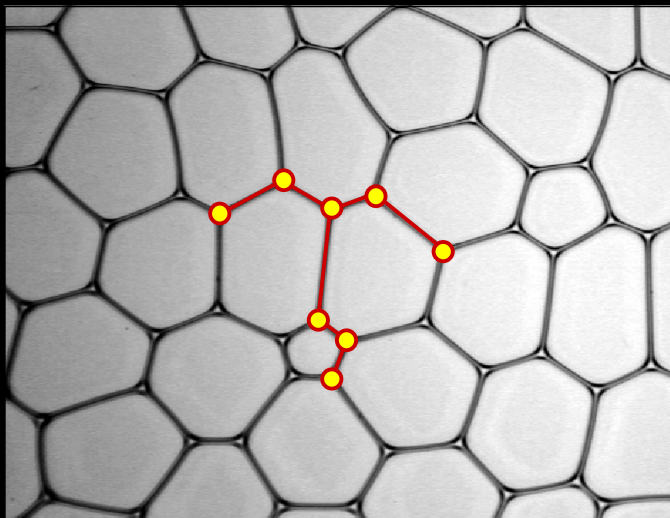
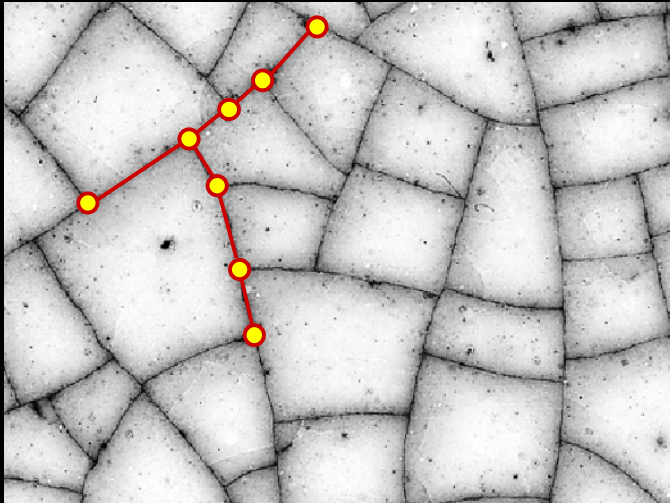
---





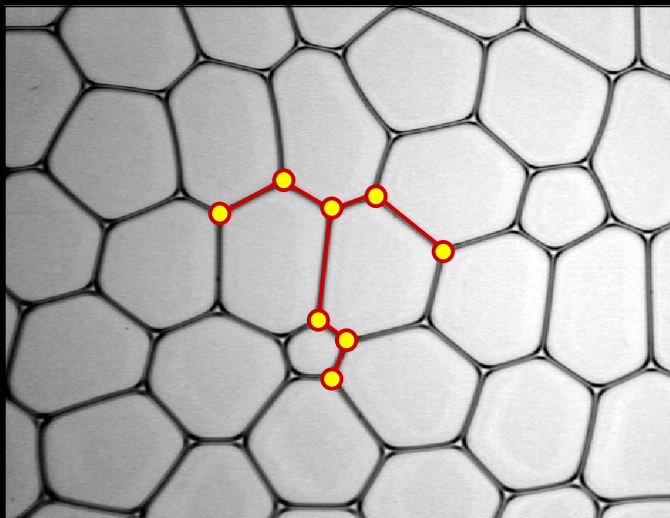
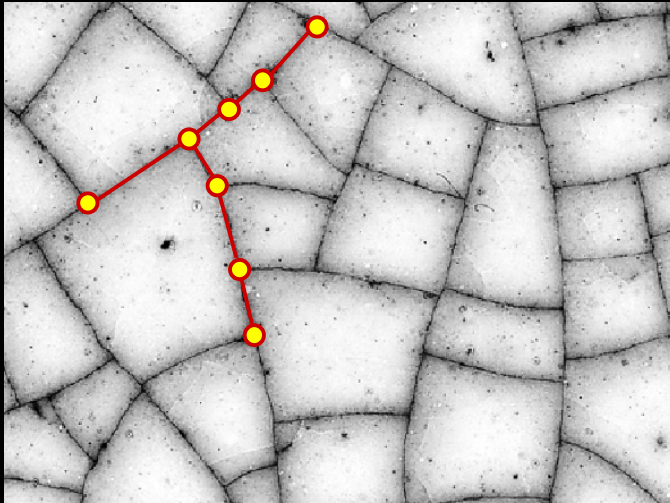
## Describe them in terms of graphs

---



# The topology is similar

---



Euler formula states that

$$N - E + F = 2$$

From which follows that

$$E \leq 3N - 6$$

and

$$\langle k \rangle < 6$$

But here in particular:

$$\langle k \rangle \approx 3$$

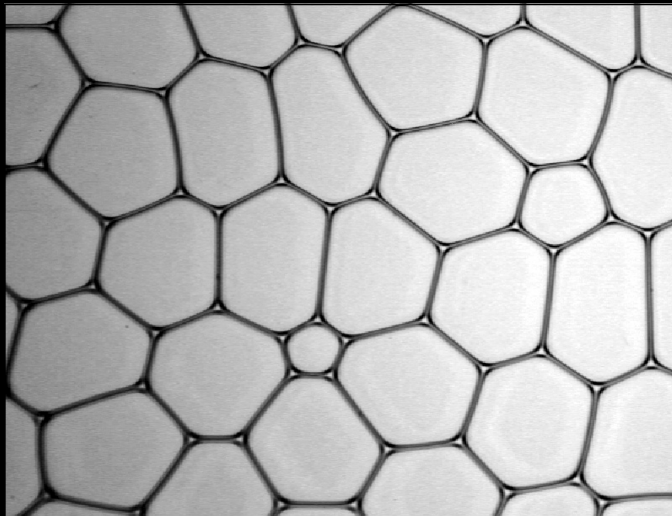
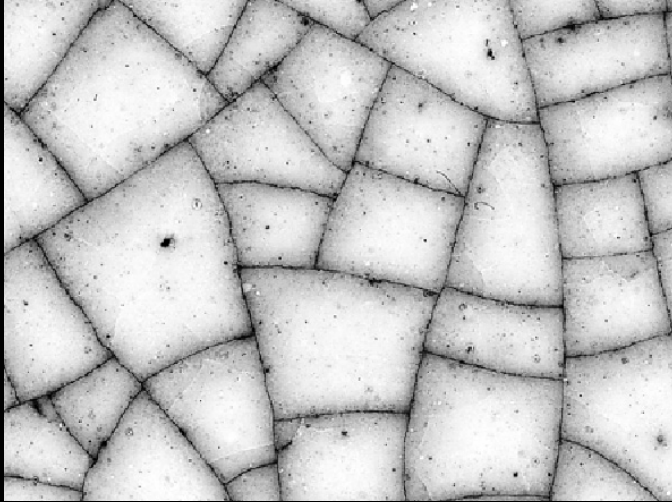
Cycles length  $\approx 6$

$$\langle L \rangle \propto \sqrt{N}$$



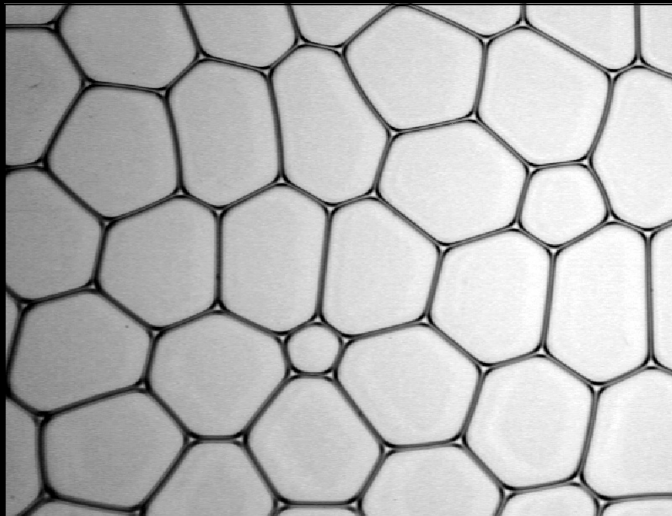
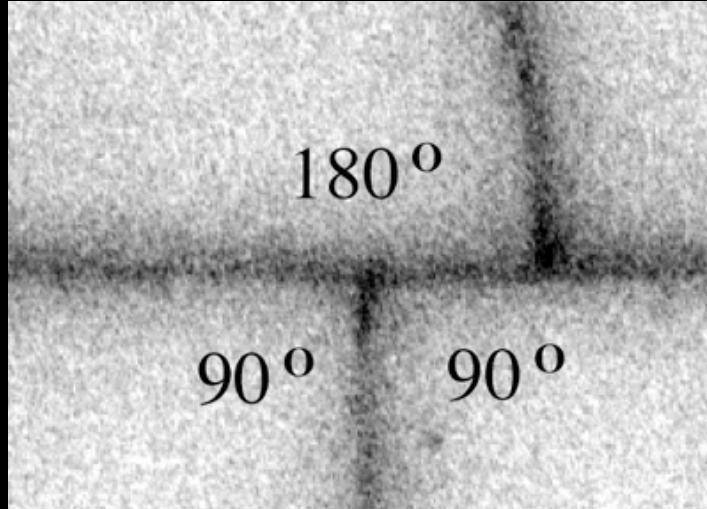
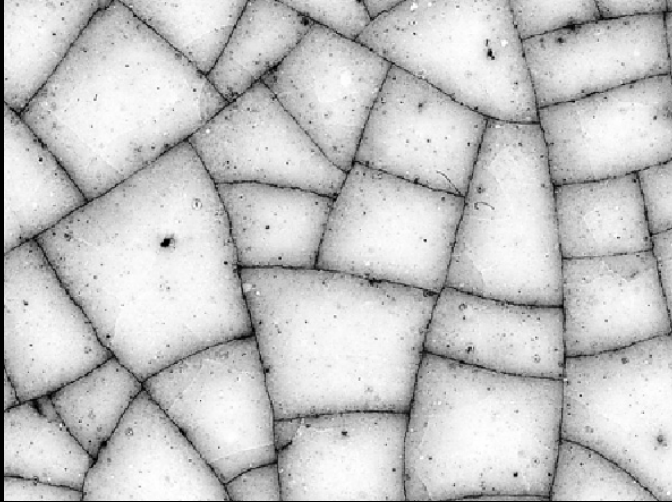
**The geometry is different**

---



# The geometry is different

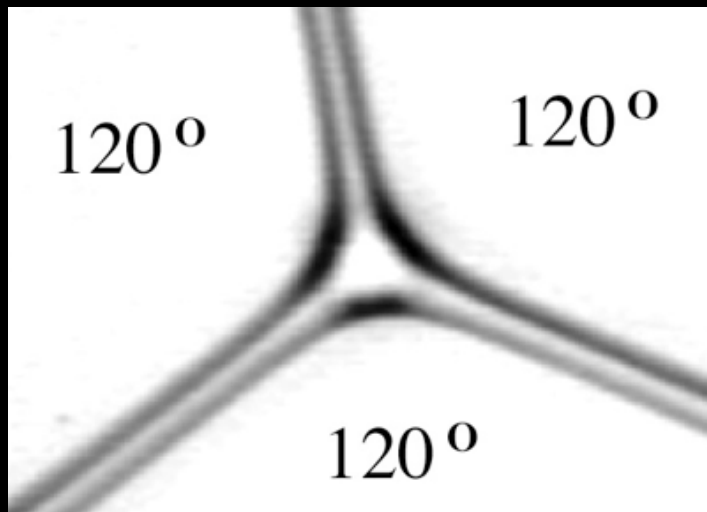
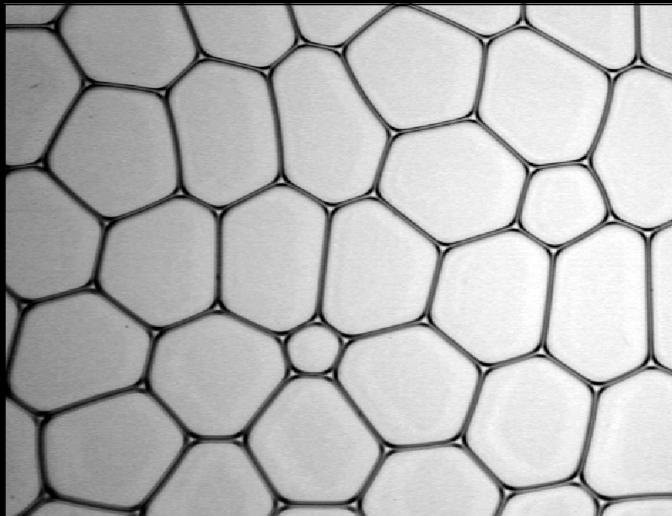
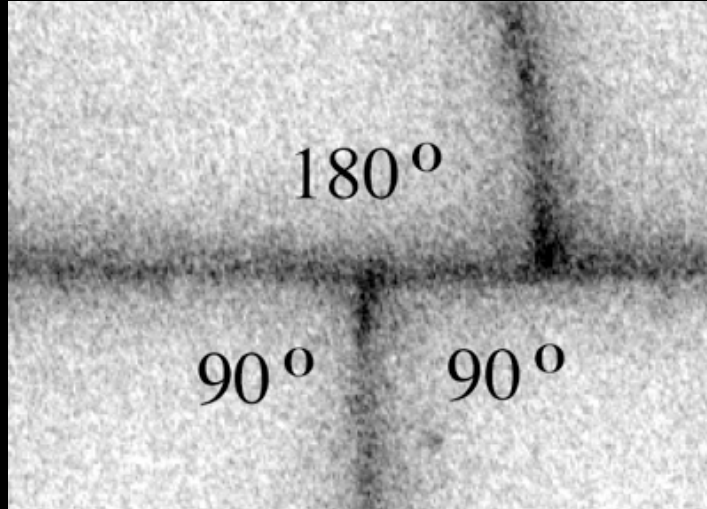
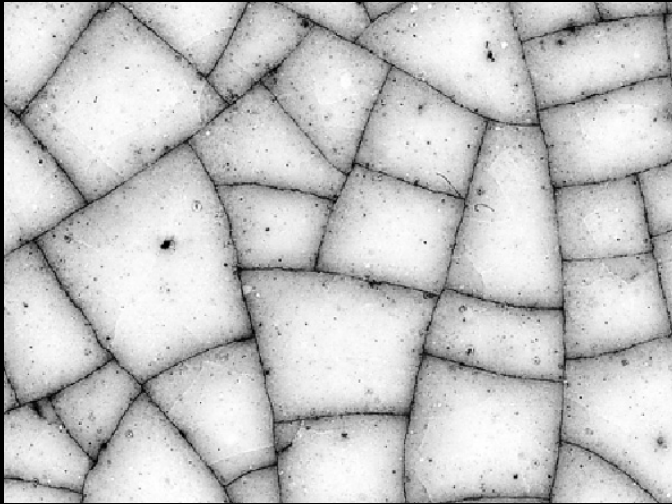
---





# The geometry is different

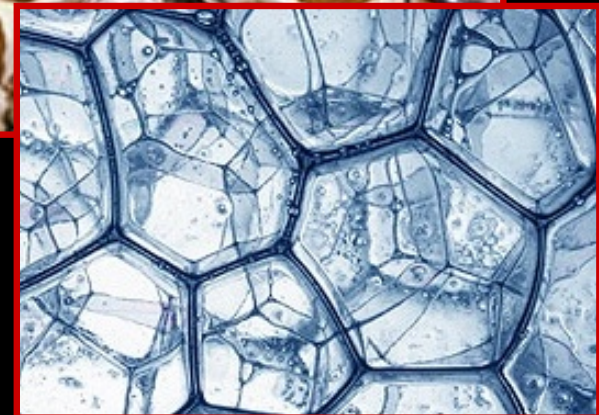
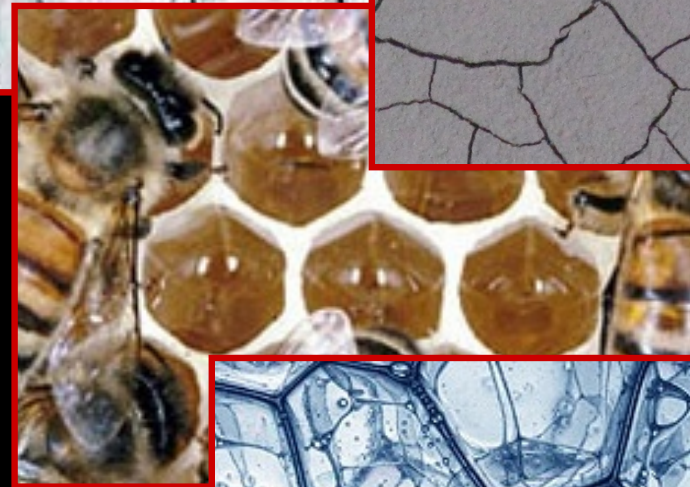
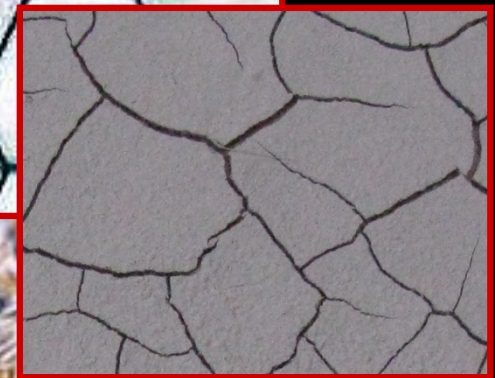
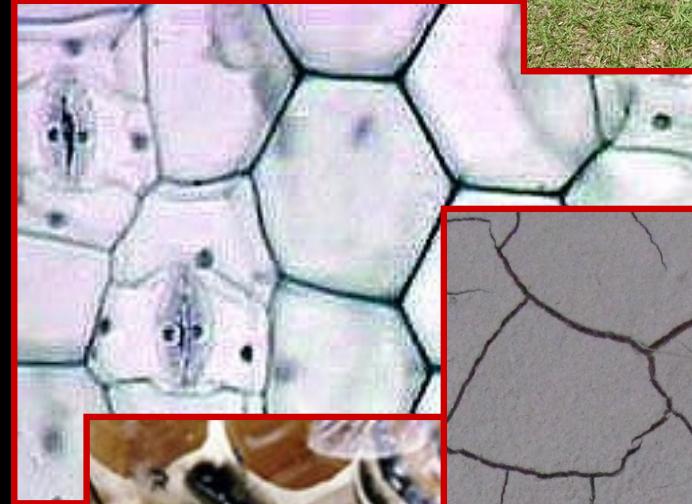
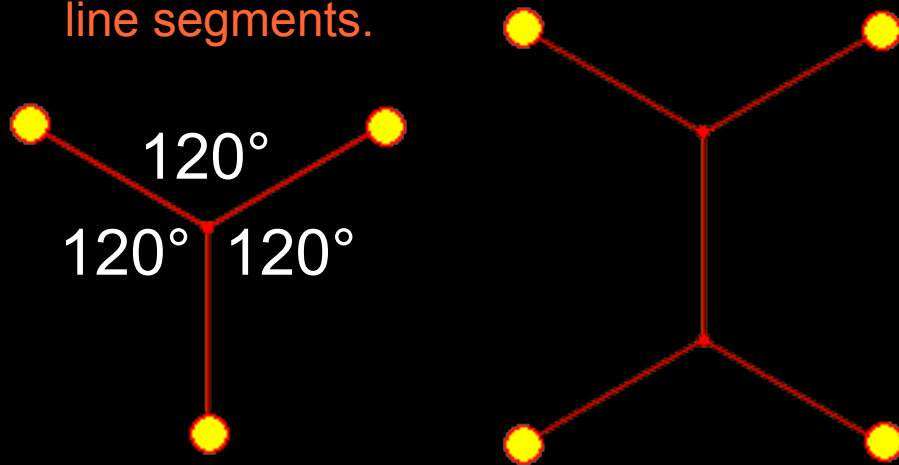
---



# Locally minimal segment length

Euclidean Steiner tree problem:

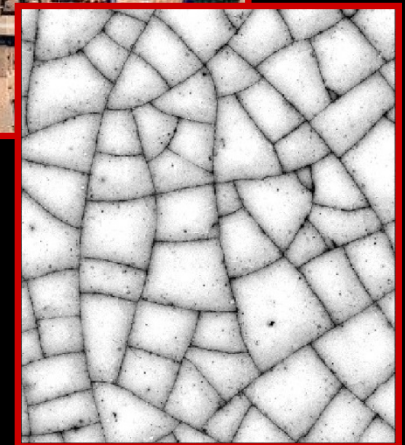
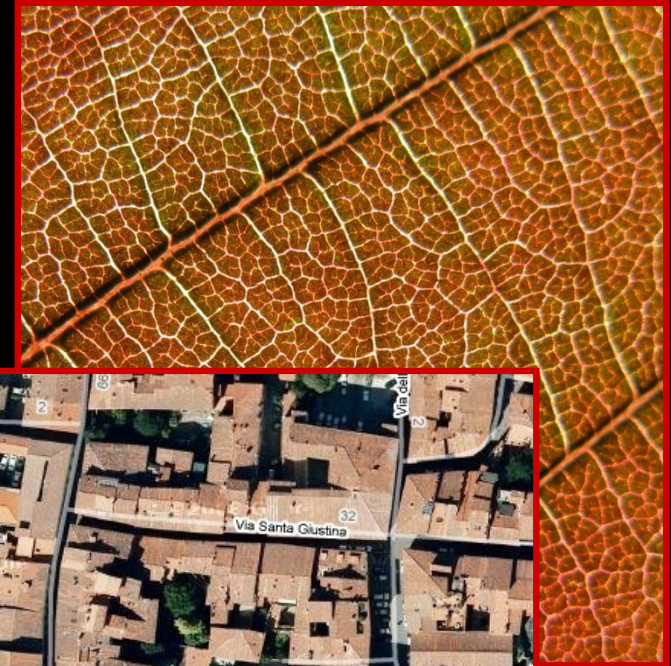
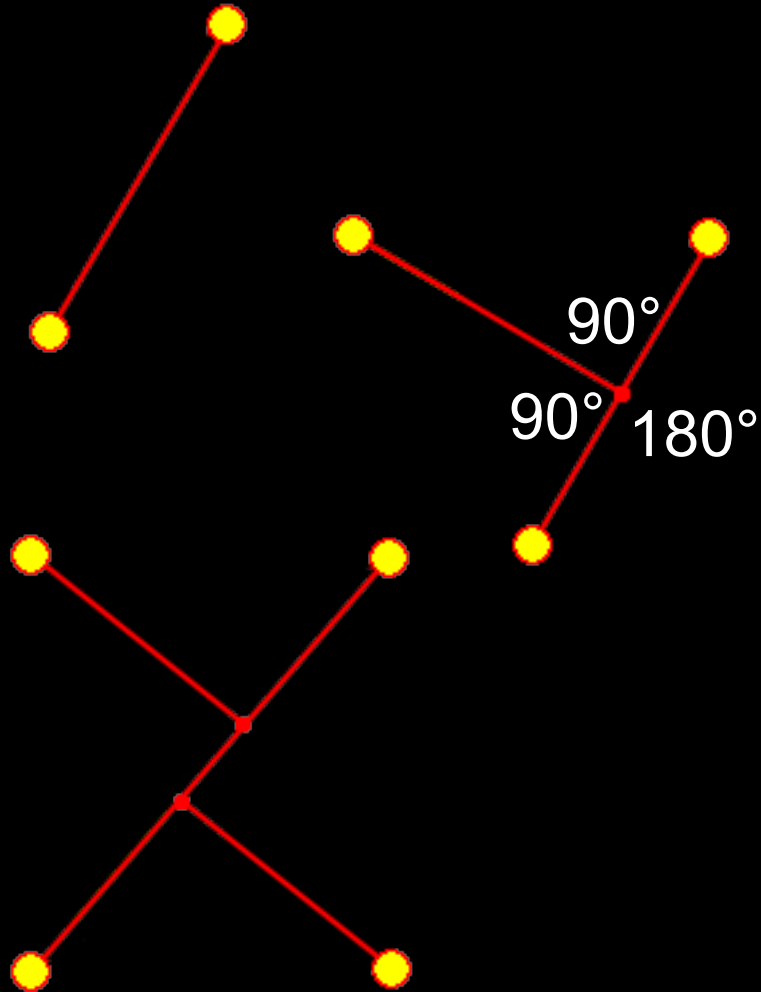
Given  $N$  points in the plane, connect them by lines of minimum total length in such a way that any two points may be interconnected by line segments either directly or via other points and line segments.





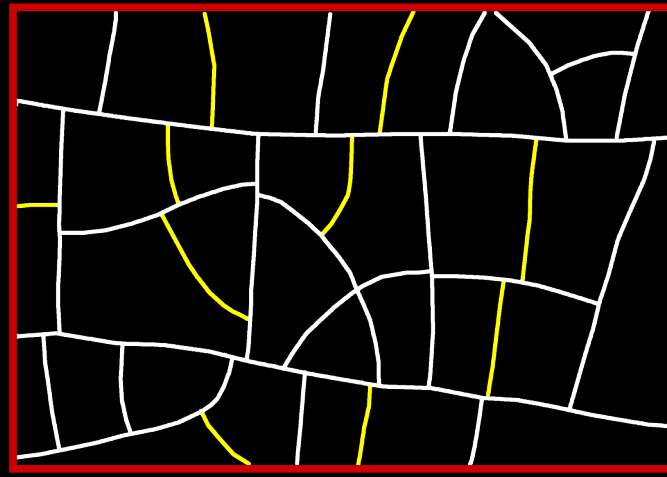
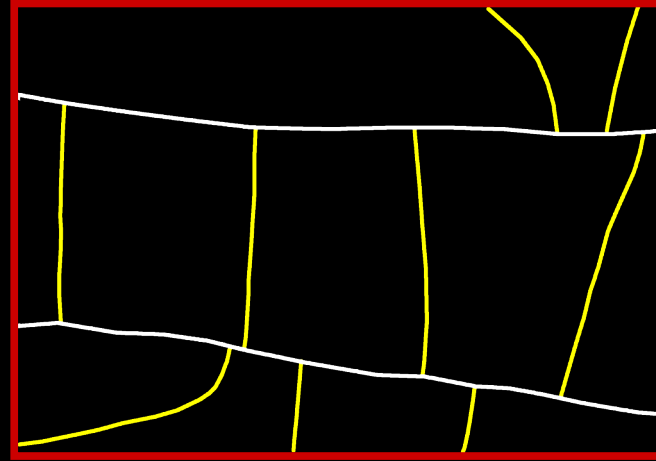
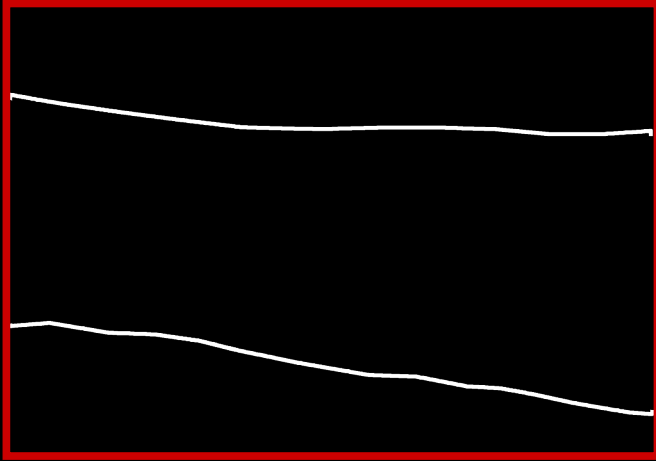
# Minimal length, but sequential construction

---



# Temporal **and** spatial **hierarchy**

---



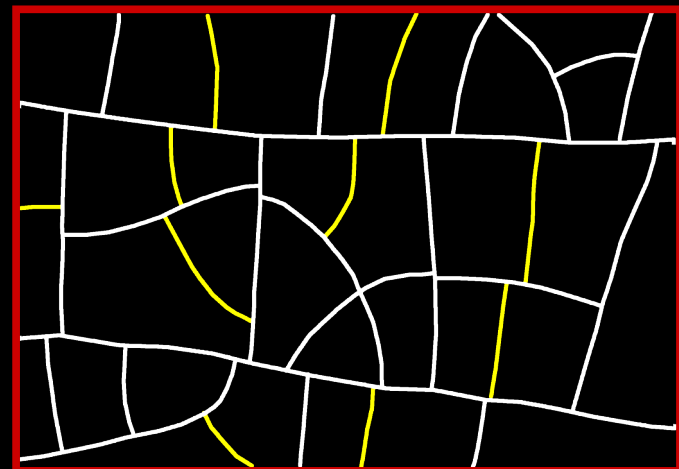
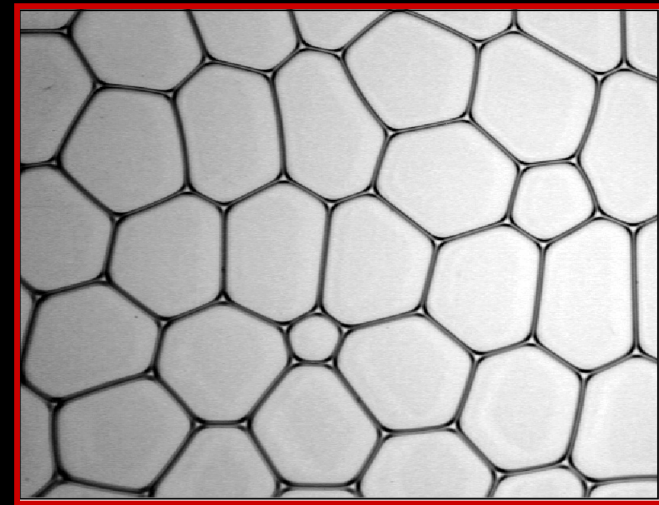


# Temporal **and** spatial **hierarchy**

---

We want a method that:

- 1) Gives different description of “hierarchical” and “equilibrium” patterns (e.g. of froths and ceramics fractures)

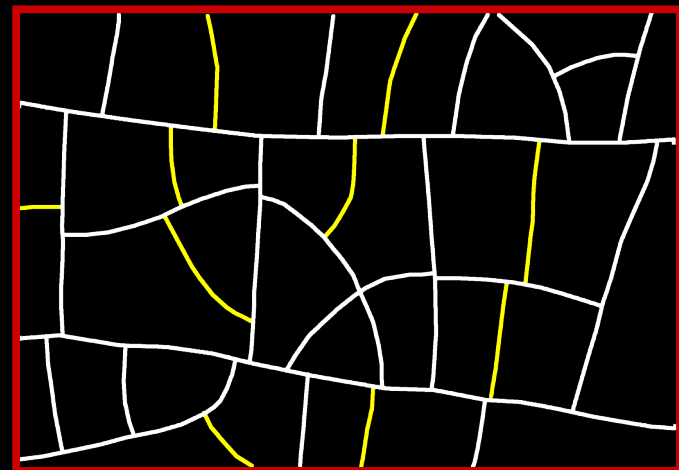
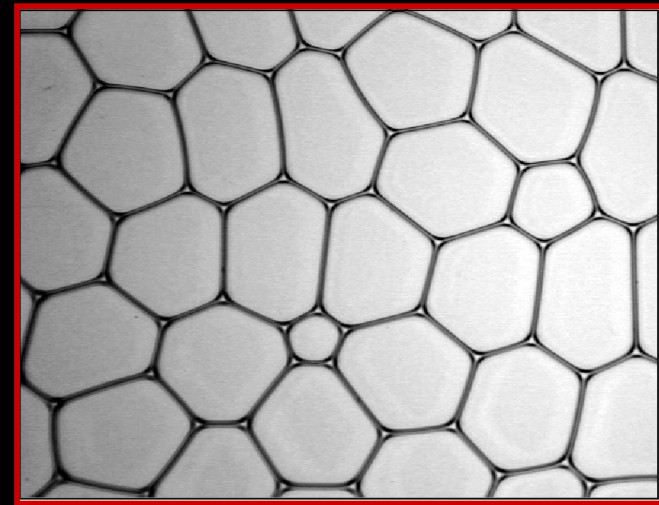


# Temporal **and** spatial **hierarchy**

---

We want a method that:

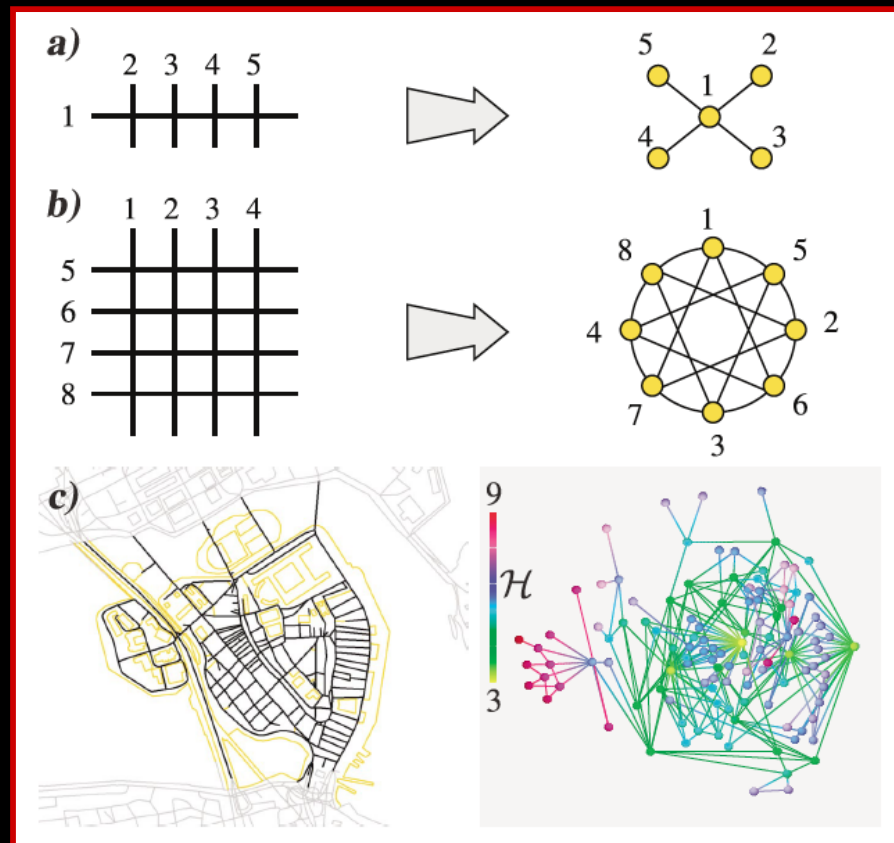
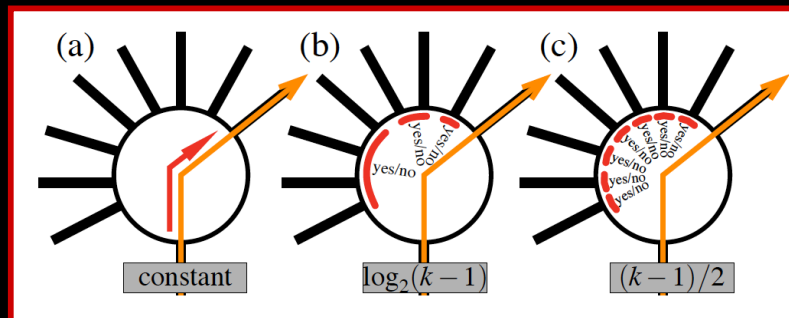
- 1) Gives different description of “hierarchical” and “equilibrium” patterns (e.g. of froths and ceramics fractures)
- 2) Provides some approximation of the order of appearance of **network segments**





# Geometry of spatial networks, a review

- 1) Each named street is a node



Jiang B. and Claramunt C., (2004) *Environment and Planning B*  
Kalapala V. et al. (2006) *Physical Review E*  
Rosvall M. et al. (2005) *Physical Review Letters*  
(...)

# Geometry of spatial networks, a review

---

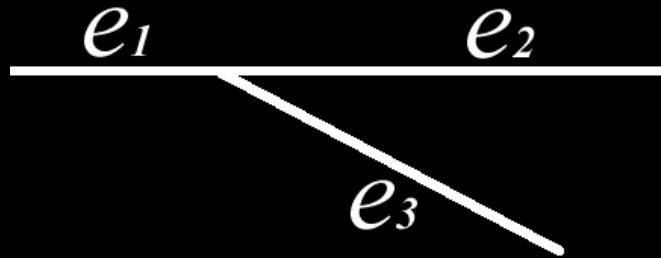
- 1) Each named street is a node
- 2) Streets defined by visual or spatial continuity

Hillier B. and Hanson J., (1984) *The Social Logic of Space*  
Hillier B. et al. (1993) *Environment and Planning B*  
Thomson R.C. (2004) *Proc. 4th Int. Space Syntax Symposium*  
Porta S. et al. (2006) *Physica A*  
(...)



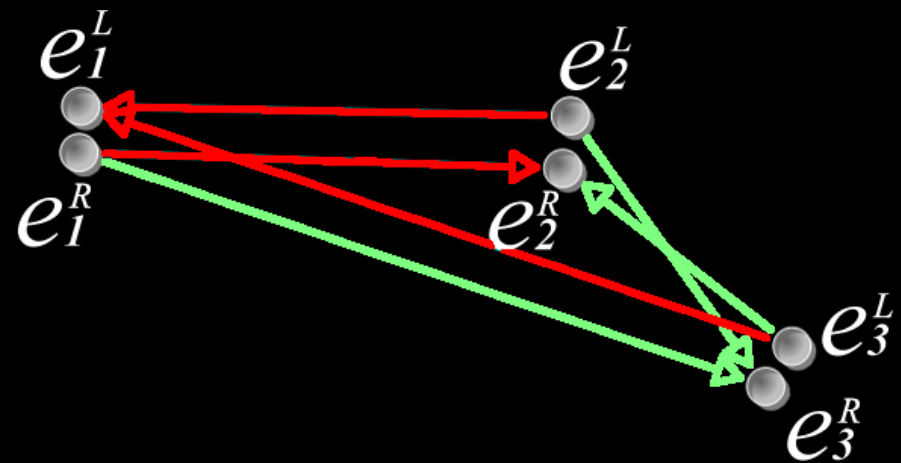
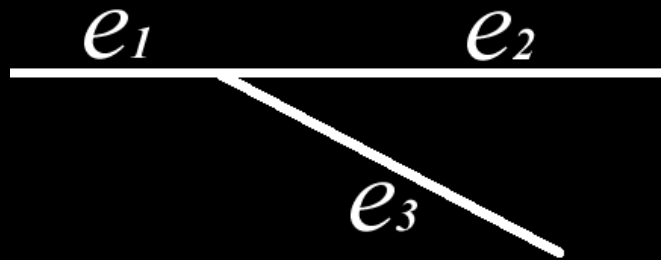
# Our approach

---





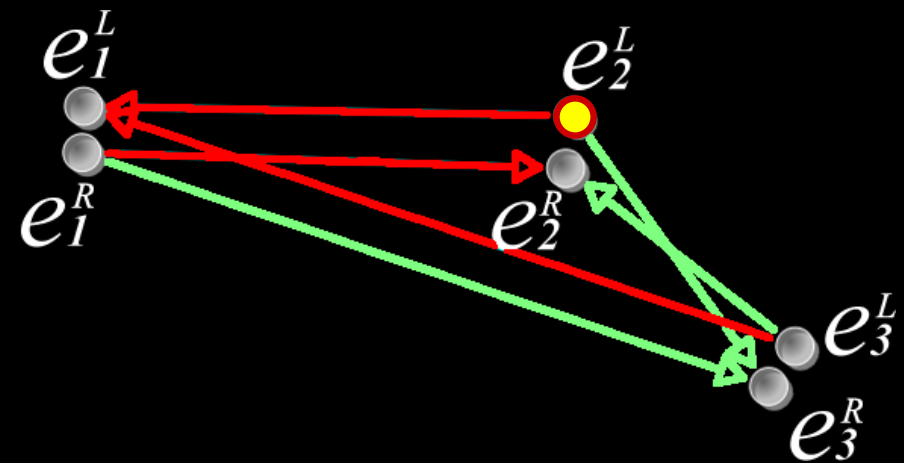
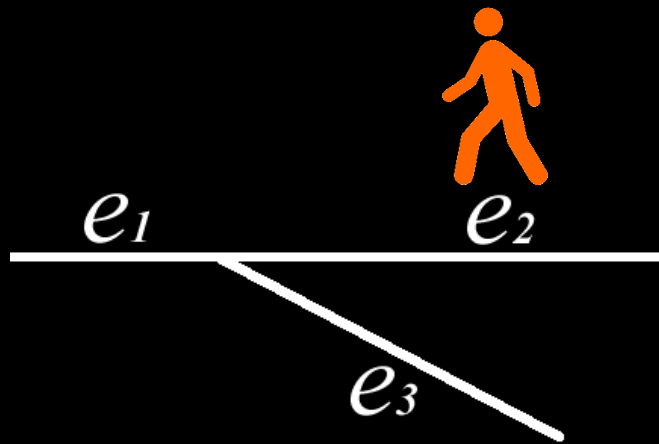
## Our approach: a weighted directed line graph



● Weight = 0

● Weight = 1

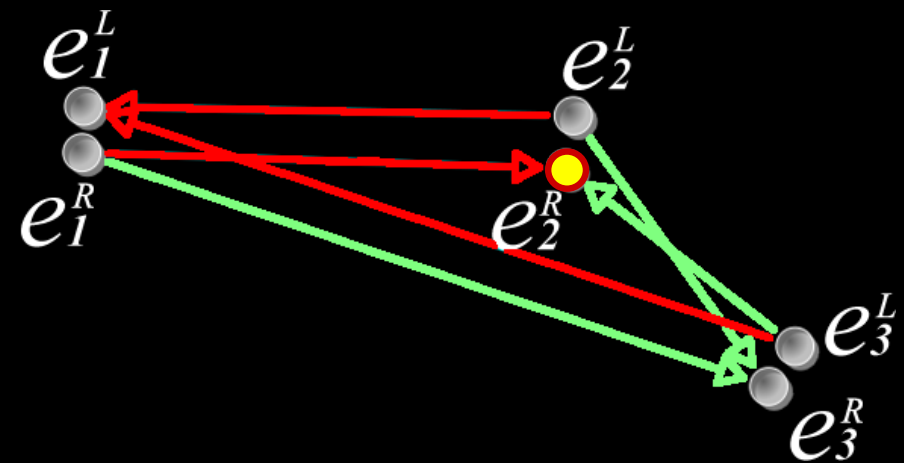
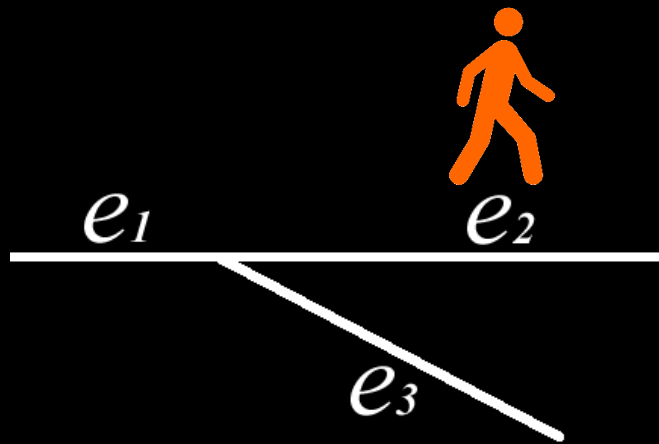
## Our approach: a weighted directed line graph



● Weight = 0

● Weight = 1

## Our approach: a weighted directed line graph

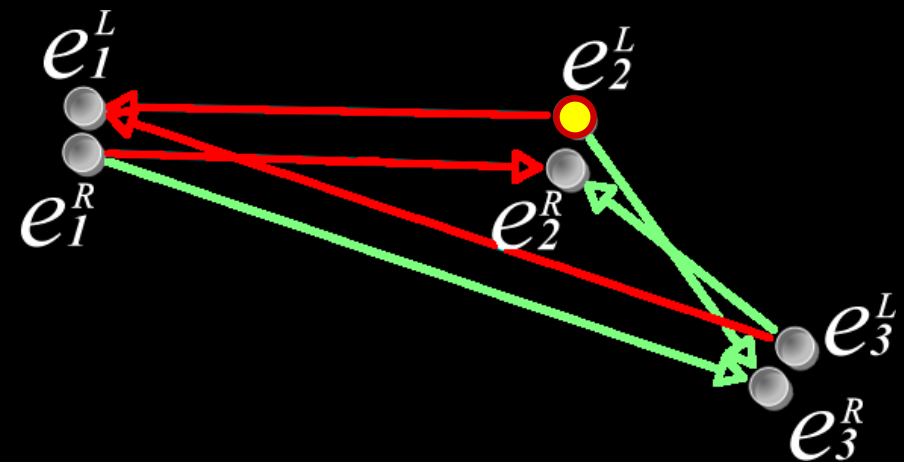
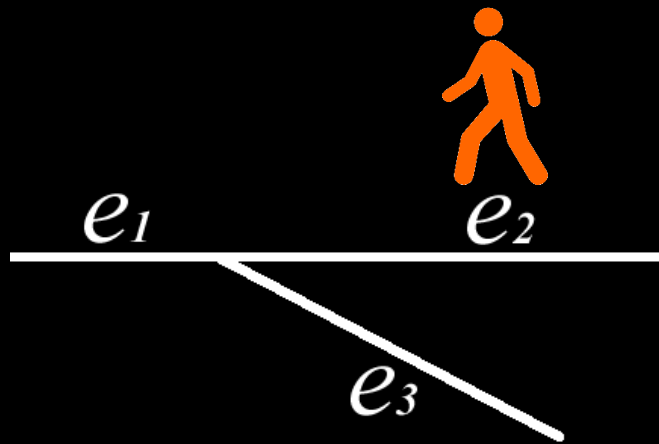


● Weight = 0

● Weight = 1



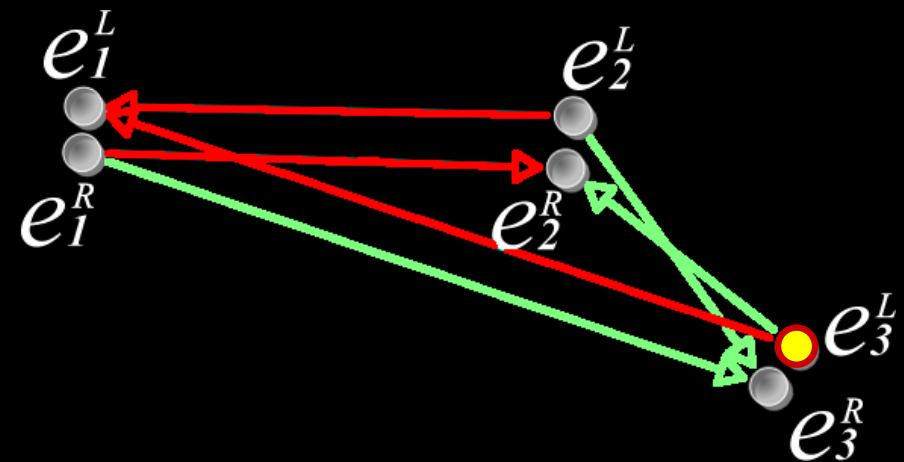
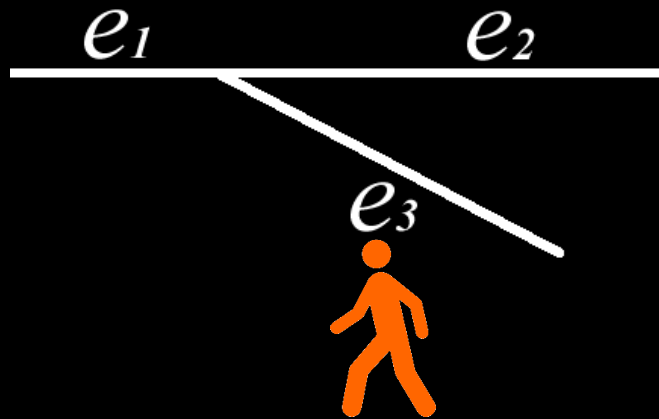
## Our approach: a weighted directed line graph



● Weight = 0

● Weight = 1

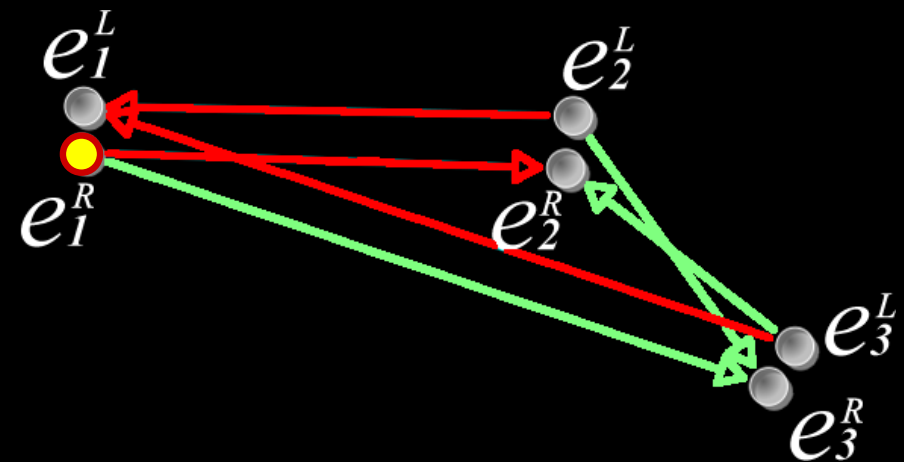
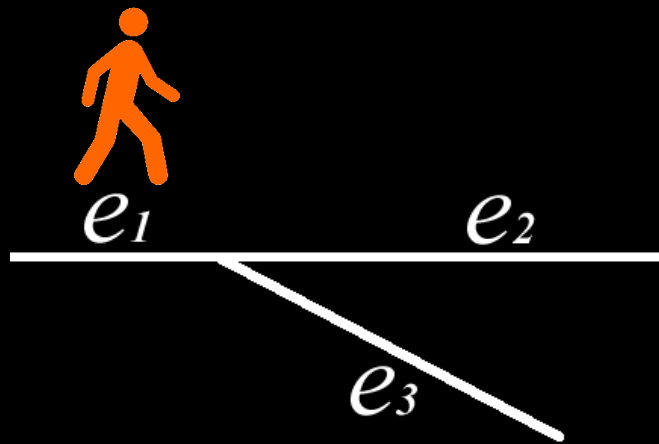
## Our approach: a weighted directed line graph



● Weight = 0

● Weight = 1

## Our approach: a weighted directed line graph



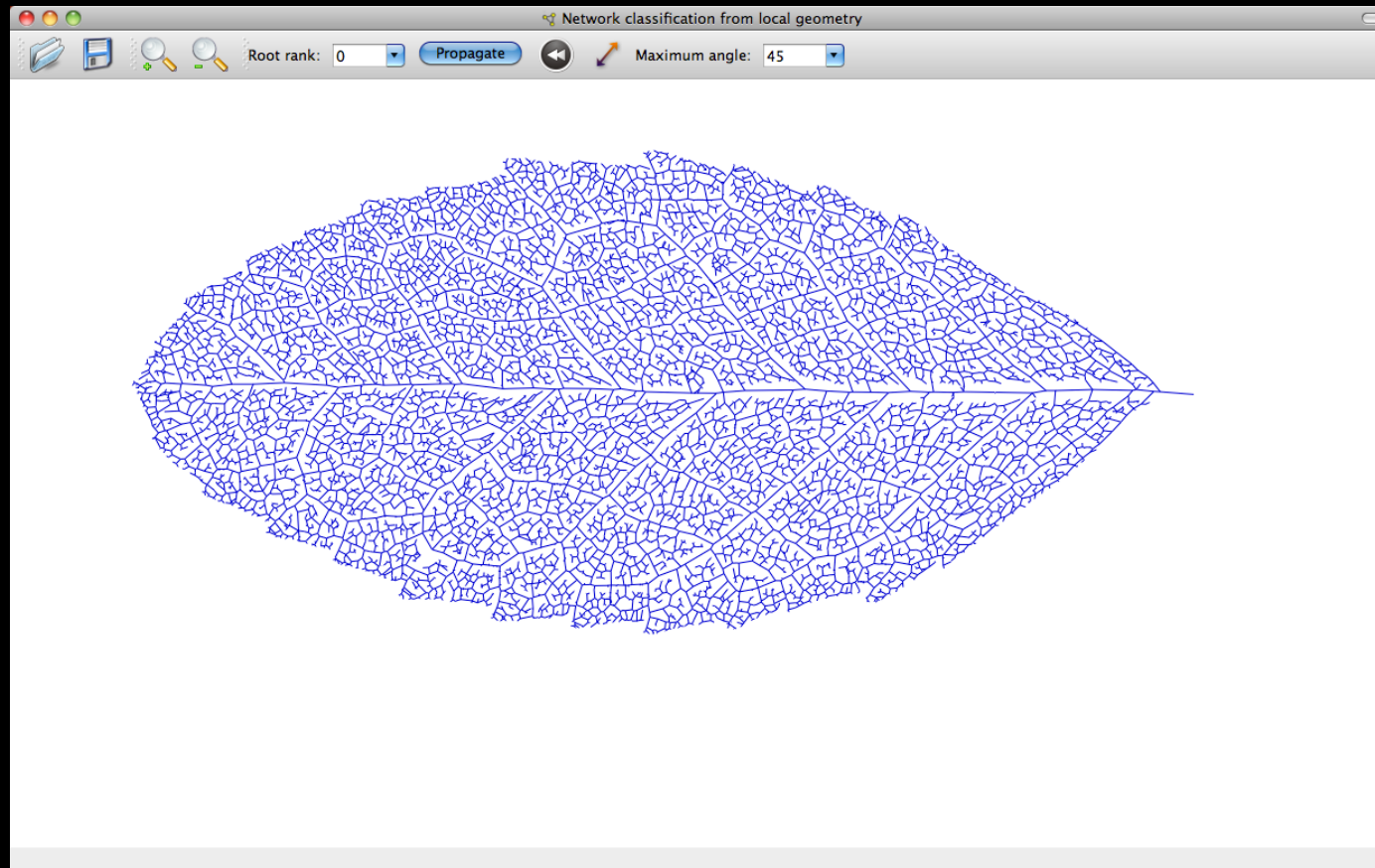
● Weight = 0

● Weight = 1



# Let's try!

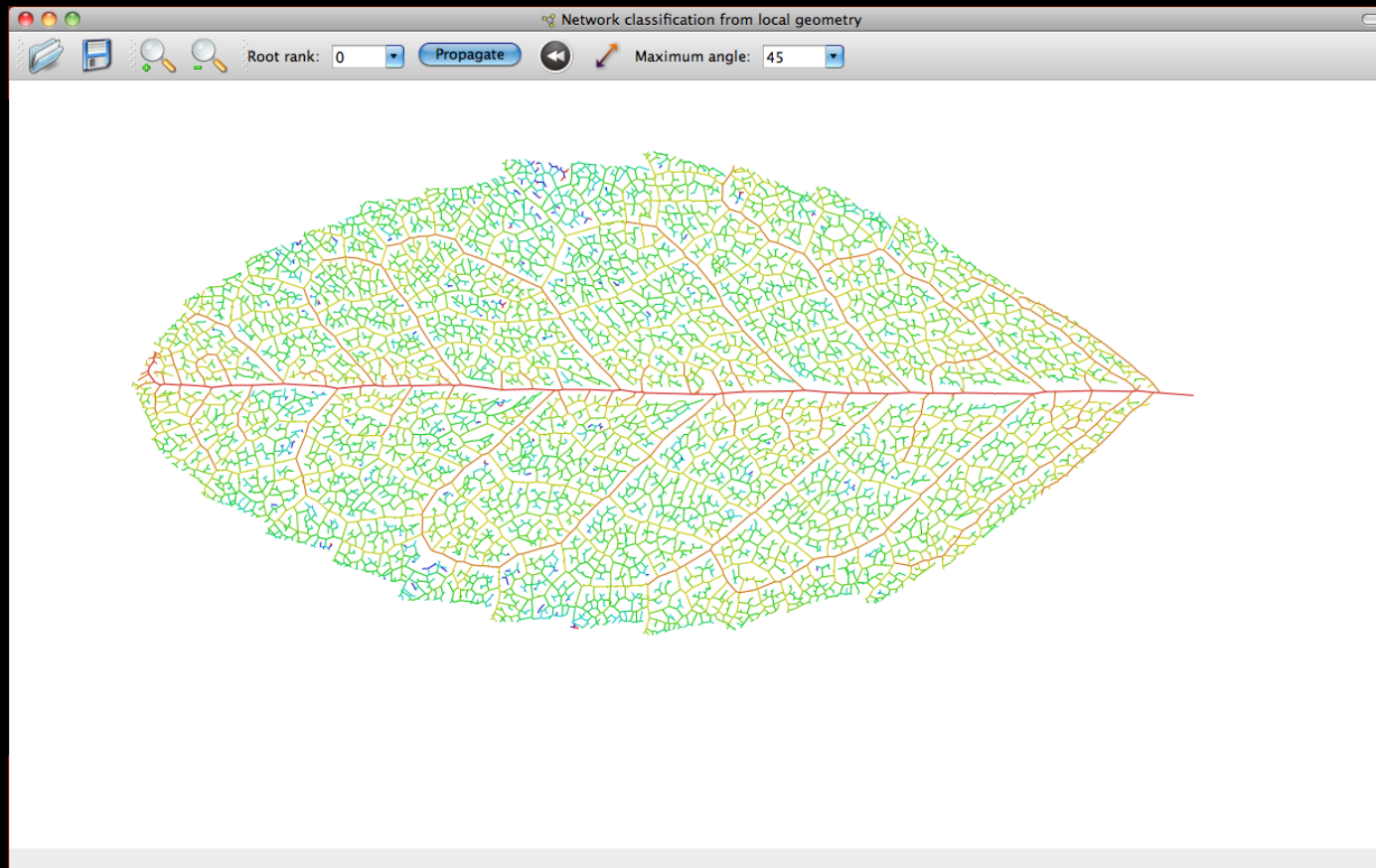
---



Free download from <http://dx.doi.org/10.1103/PhysRevE.83.066106>

# Let's try!

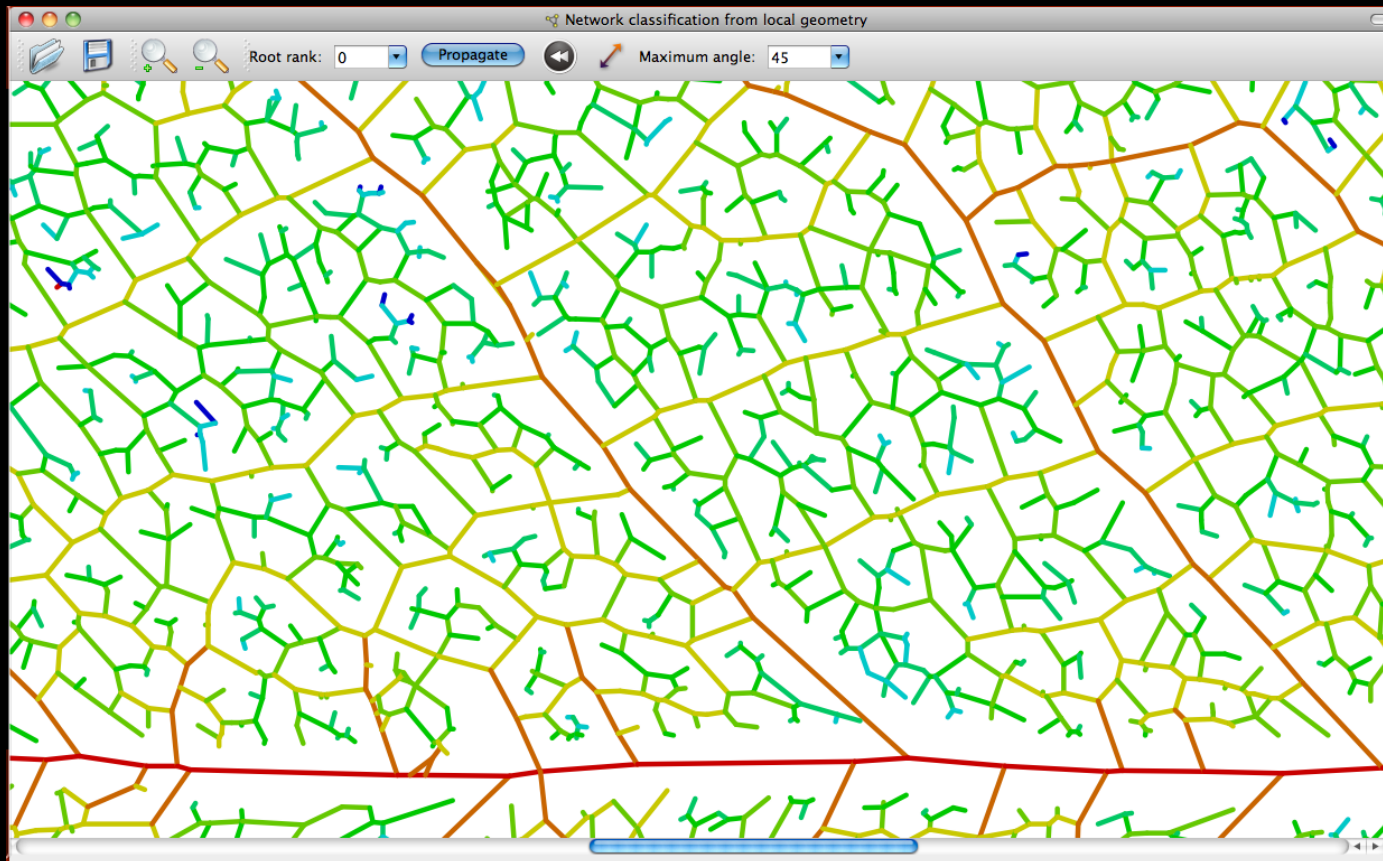
---



Free download from <http://dx.doi.org/10.1103/PhysRevE.83.066106>

# Let's try!

---



Free download from <http://dx.doi.org/10.1103/PhysRevE.83.066106>



# Inferred orders **vs.** “age”

---



The diameter of leaf veins is roughly proportional to vein age (because veins keep growing continuously since they first appear)

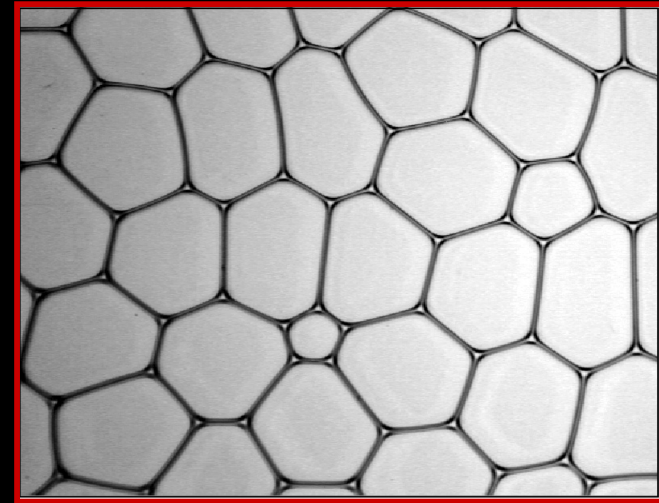
We can compare the rank assigned to veins by our algorithm (which is based only on junction angles, not on the size of the veins) with the vein diameter to assess to what extent the classification recovers the temporal information.

# Temporal **and** spatial **hierarchy**

---

We want a method that:

- 1) Gives different description of “hierarchical” and “equilibrium” patterns (of froths and ceramics fractures)
- 2) Provides some approximation of the order of appearance of network segments



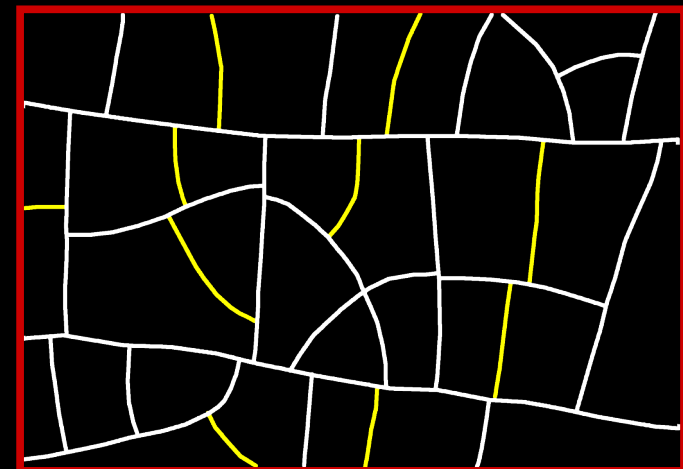
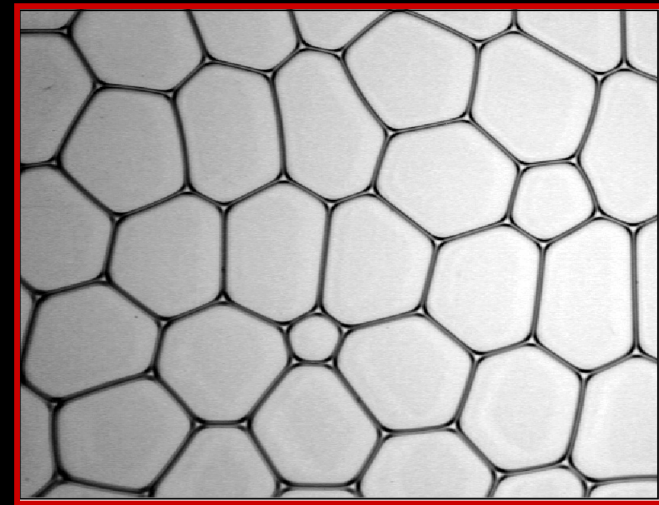
# Temporal **and** spatial **hierarchy**

---

We want a method that:

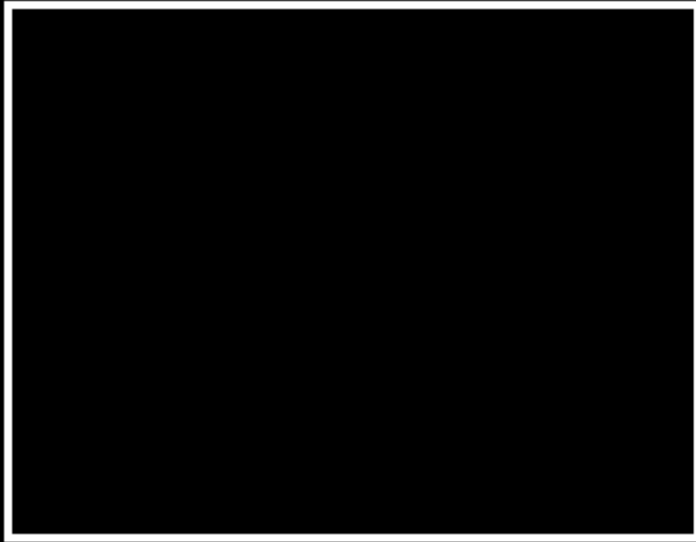
- 1) Gives different description of “hierarchical” and “equilibrium” patterns (of froths and ceramics fractures)
- 2) Provides some approximation of the order of appearance of network segments

*OK!  
but not perfectly*



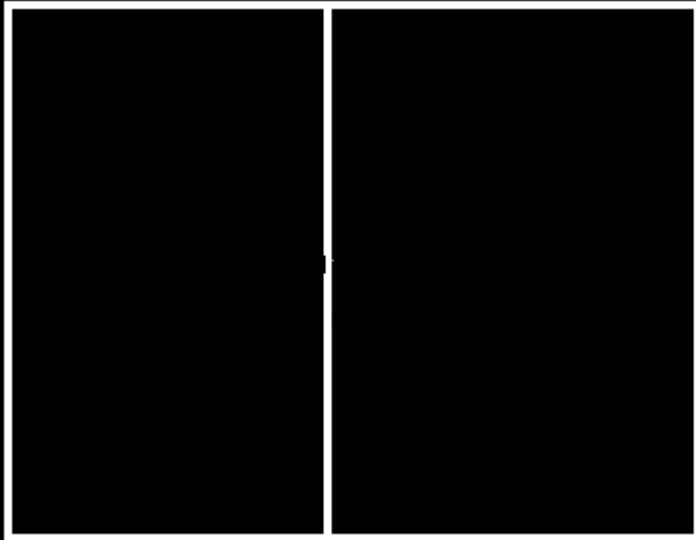


# Lattice models for hierarchical networks



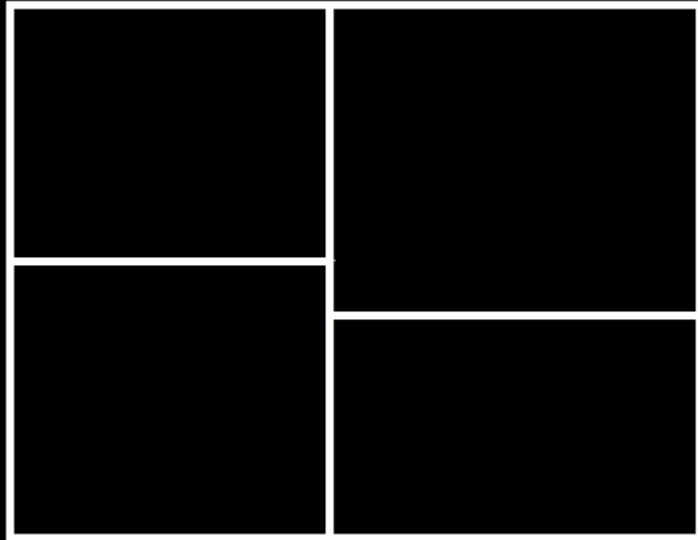
Introducing “Mondrian” lattices

# Lattice models for hierarchical networks



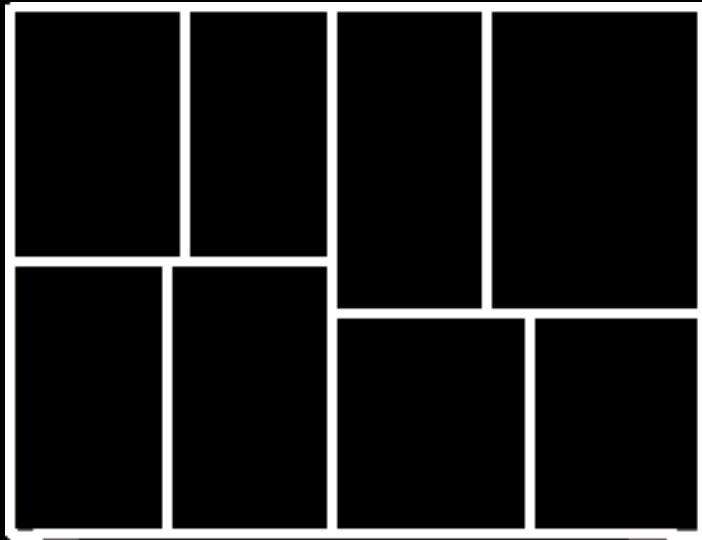
Introducing “Mondrian” lattices

# Lattice models for hierarchical networks



Introducing “Mondrian” lattices

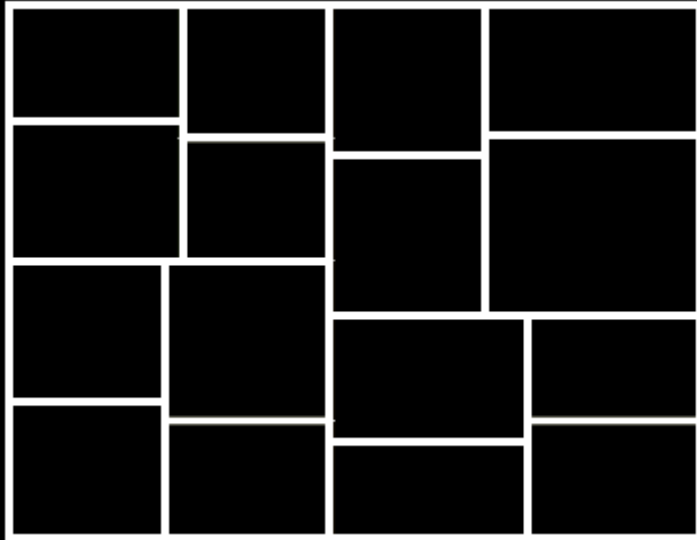
# Lattice models for hierarchical networks



Introducing “Mondrian” lattices

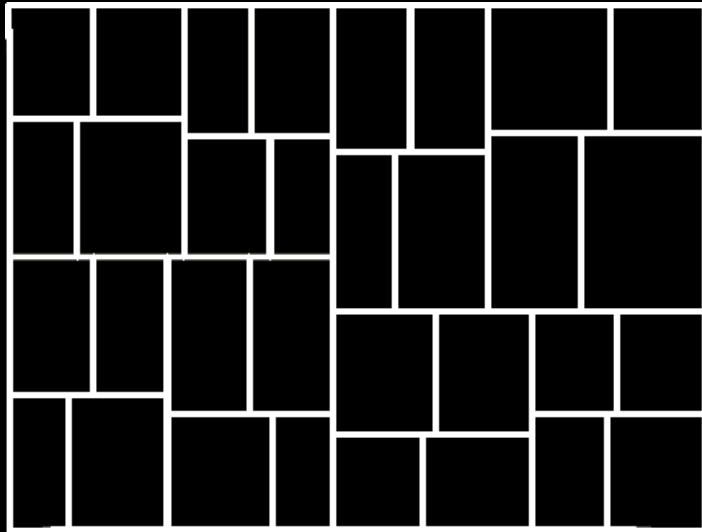


# Lattice models for hierarchical networks



Introducing “Mondrian” lattices

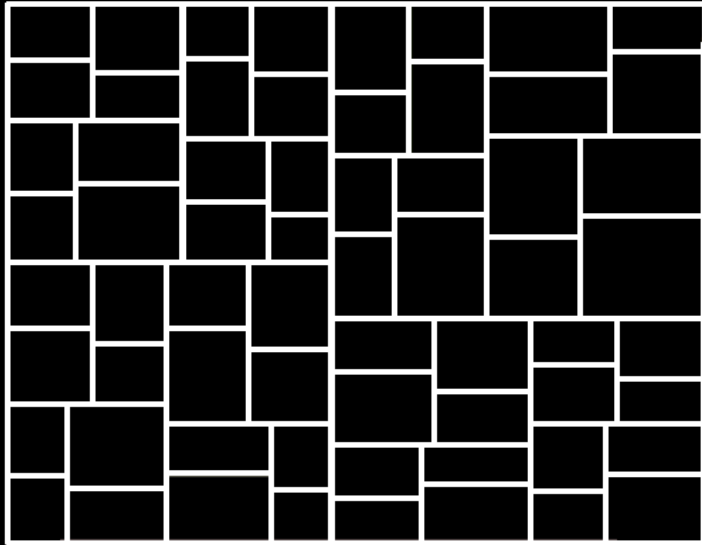
# Lattice models for hierarchical networks



Introducing “Mondrian” lattices

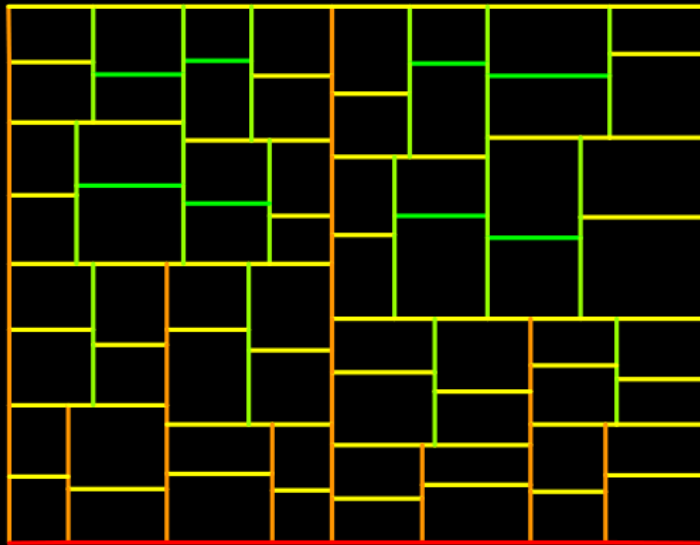
# Lattice models **for hierarchical networks**

---



Introducing “Mondrian” lattices

# Lattice models for hierarchical networks

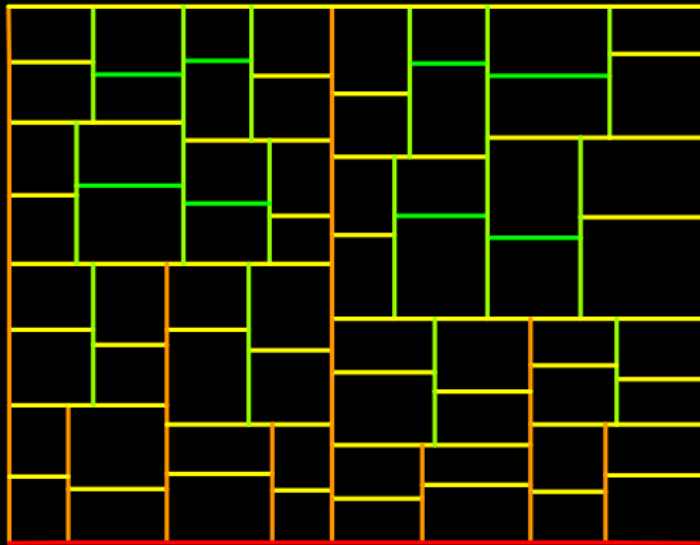


Introducing “Mondrian” lattices

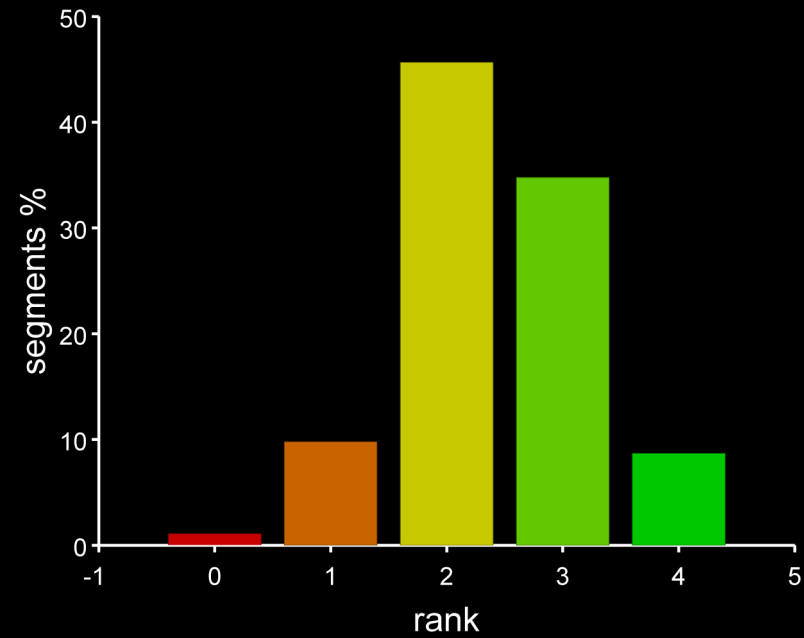


# Lattice models **for hierarchical networks**

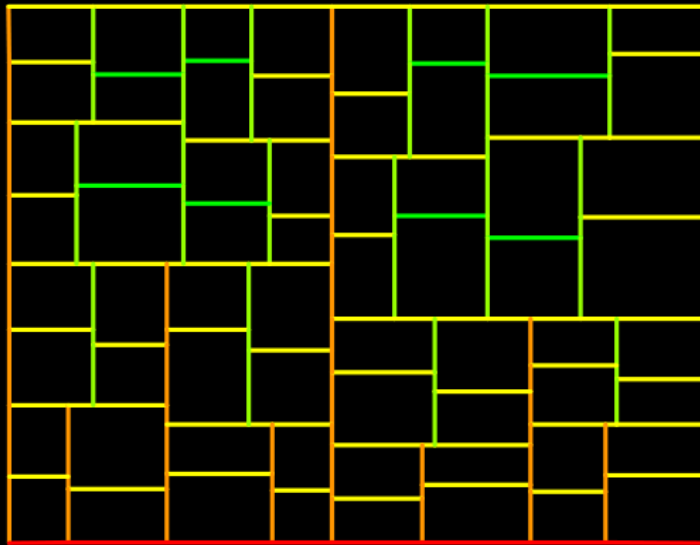
---



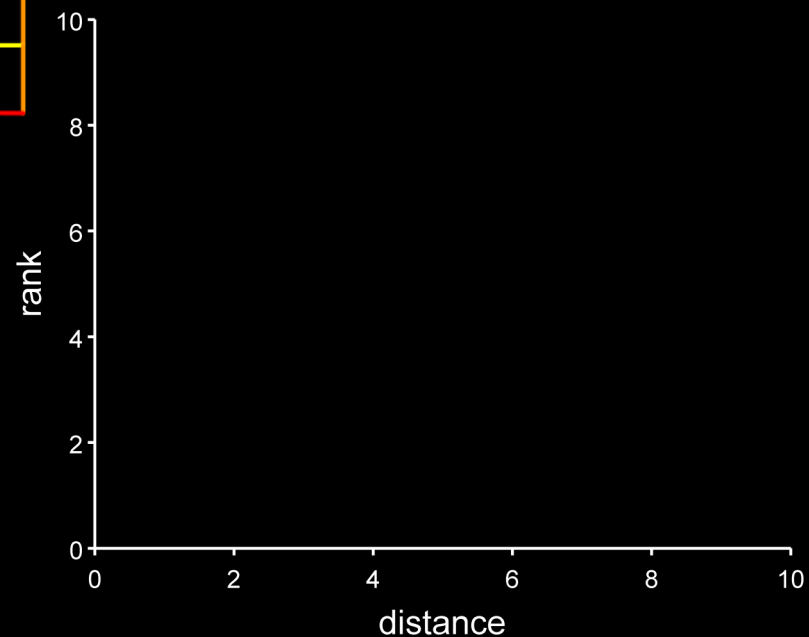
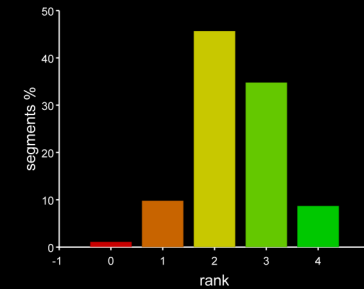
- Segment histogram



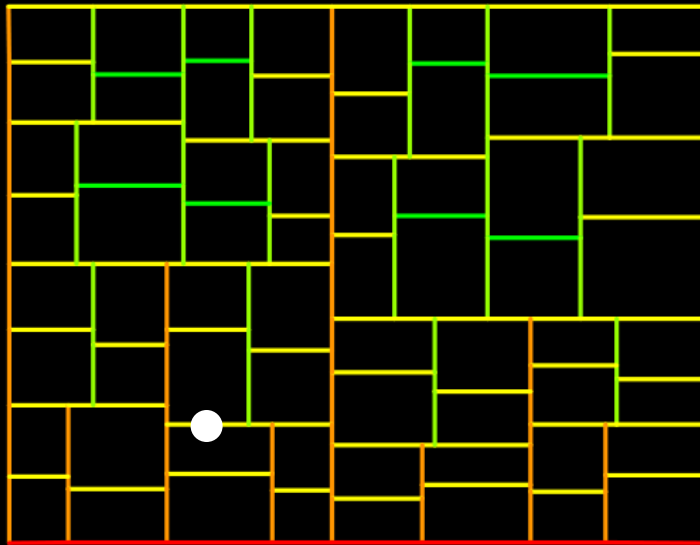
# Lattice models **for hierarchical networks**



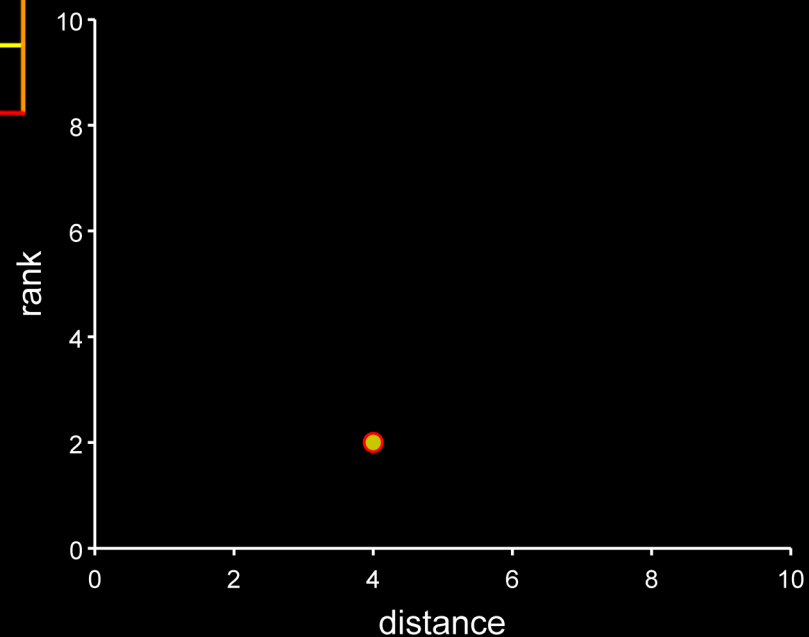
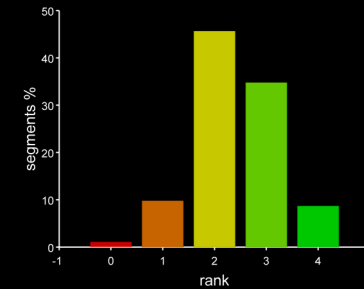
- Segment histogram
- Edge rank vs. distance



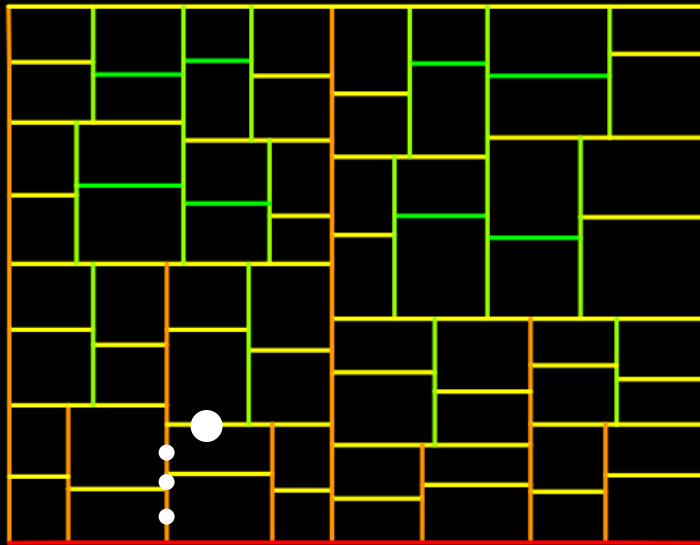
# Lattice models for hierarchical networks



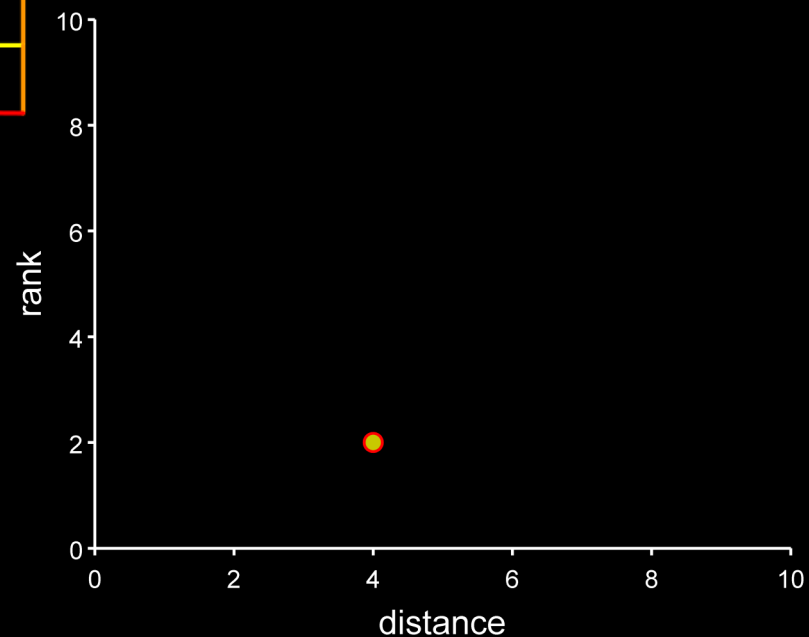
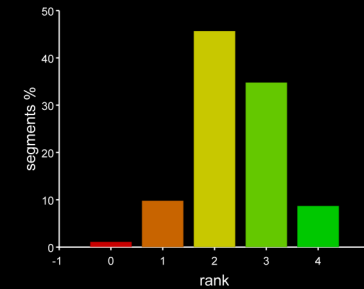
- Segment histogram
- Edge rank vs. distance



# Lattice models for hierarchical networks

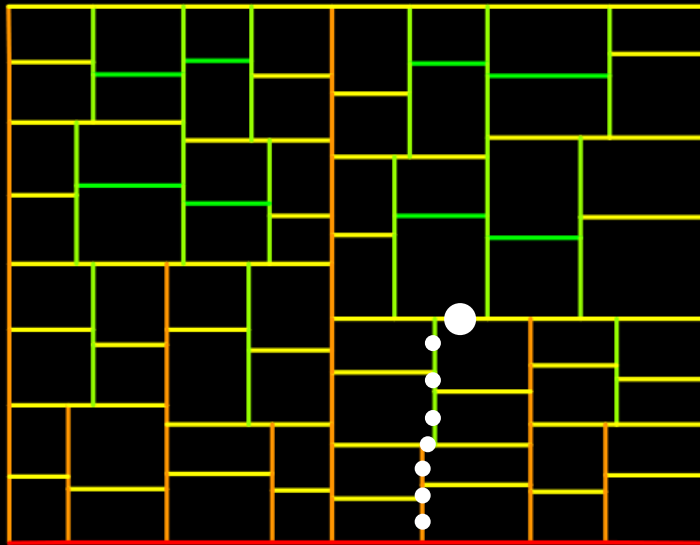


- Segment histogram
- Edge rank vs. distance

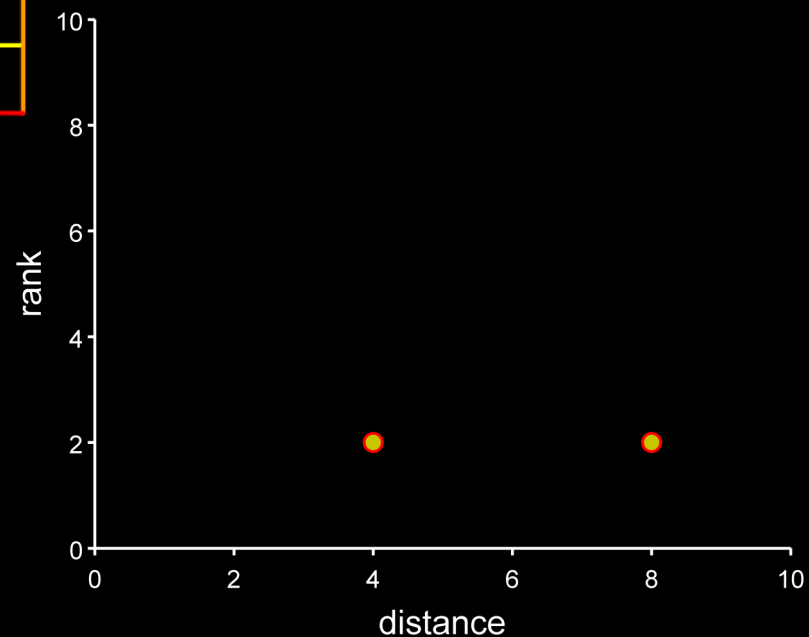
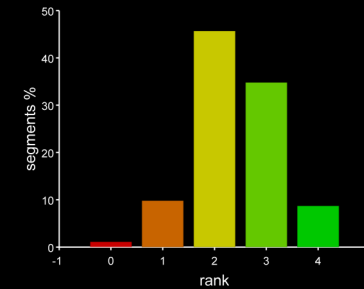




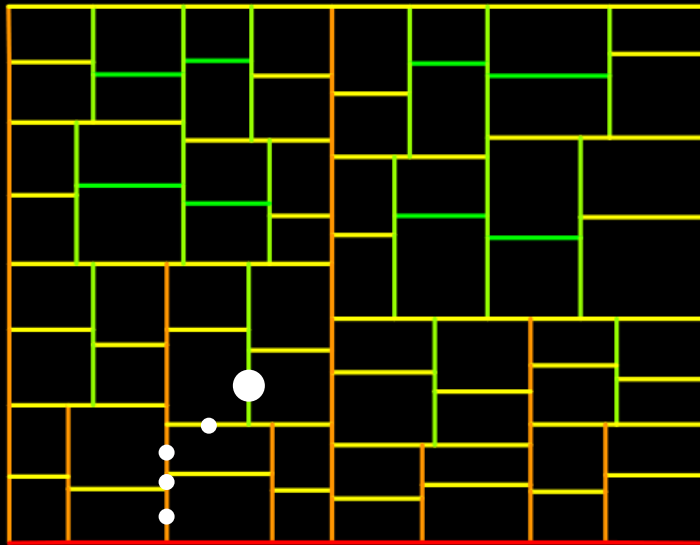
# Lattice models **for hierarchical networks**



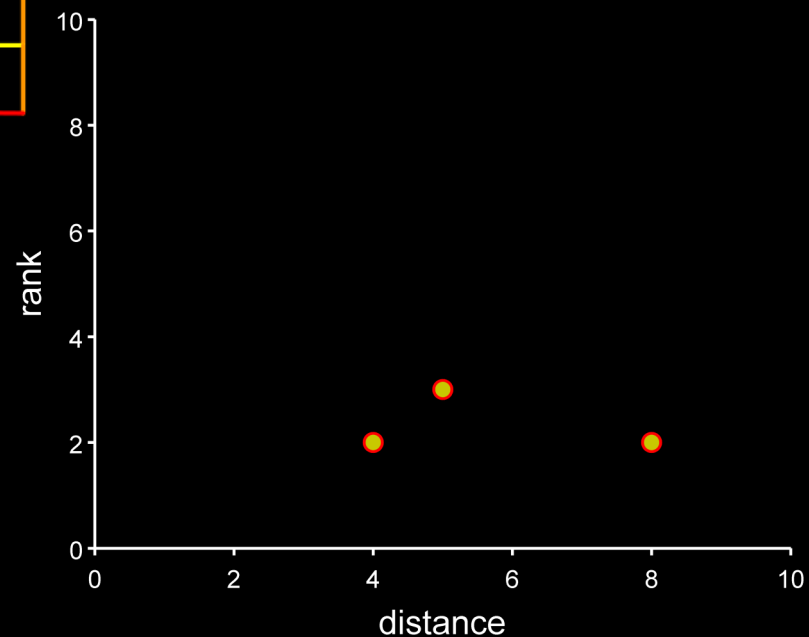
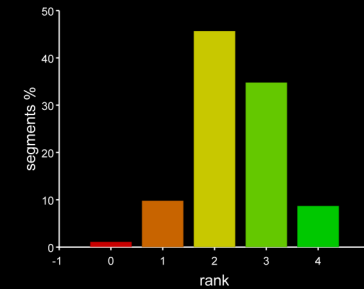
- Segment histogram
- Edge rank vs. distance



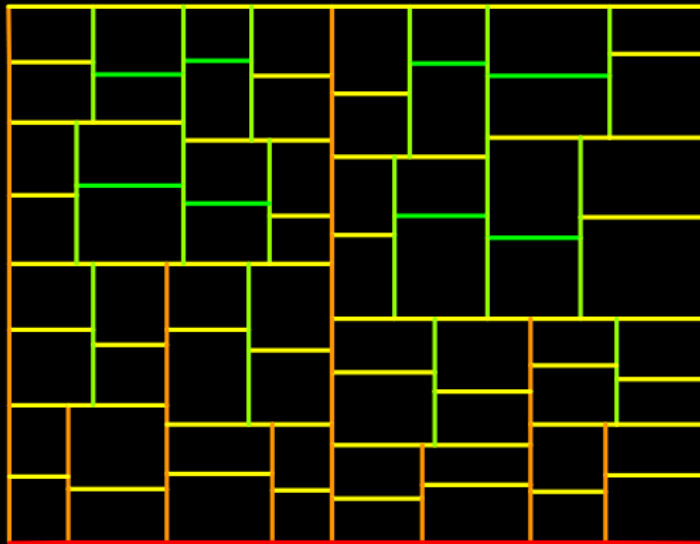
# Lattice models for hierarchical networks



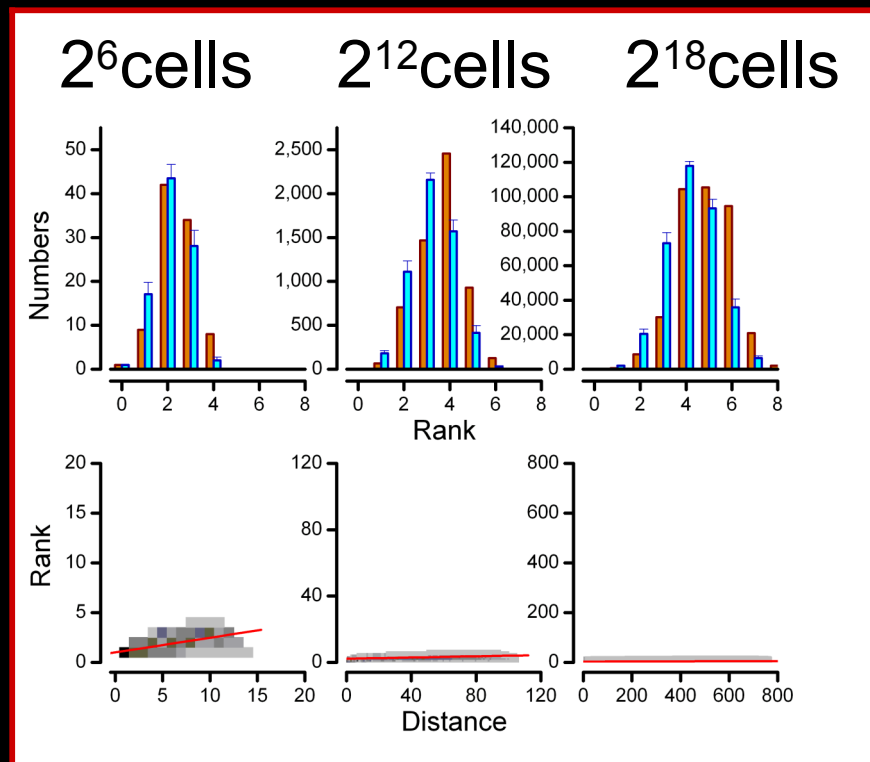
- Segment histogram
- Edge rank vs. distance



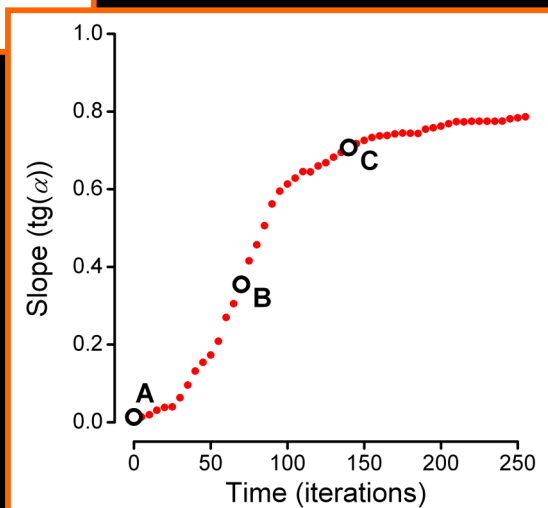
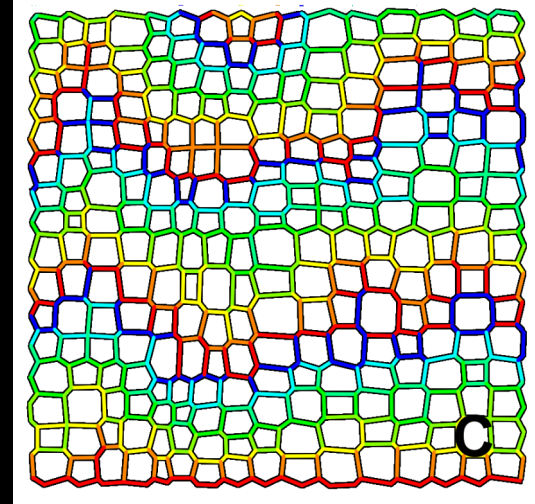
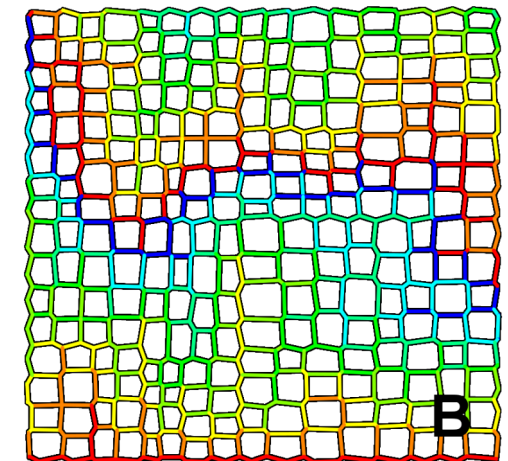
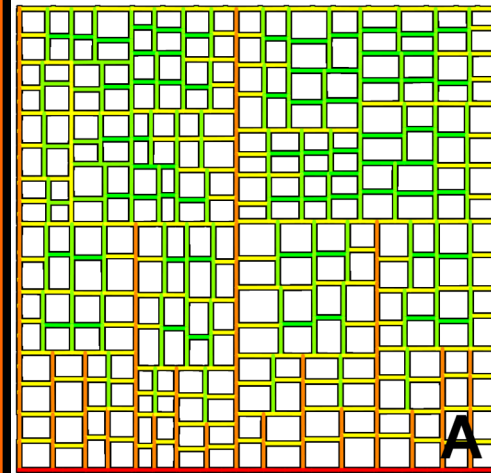
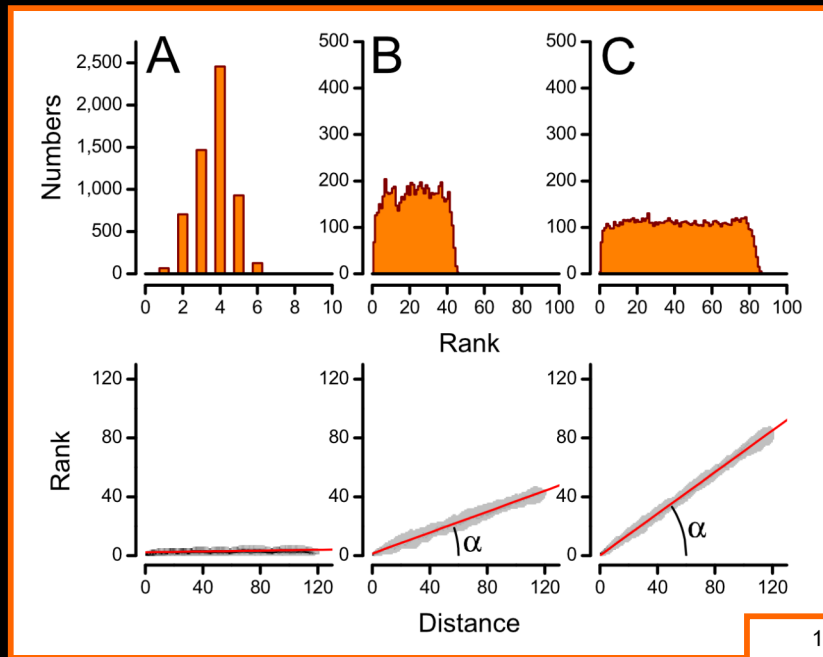
# Lattice models for hierarchical networks



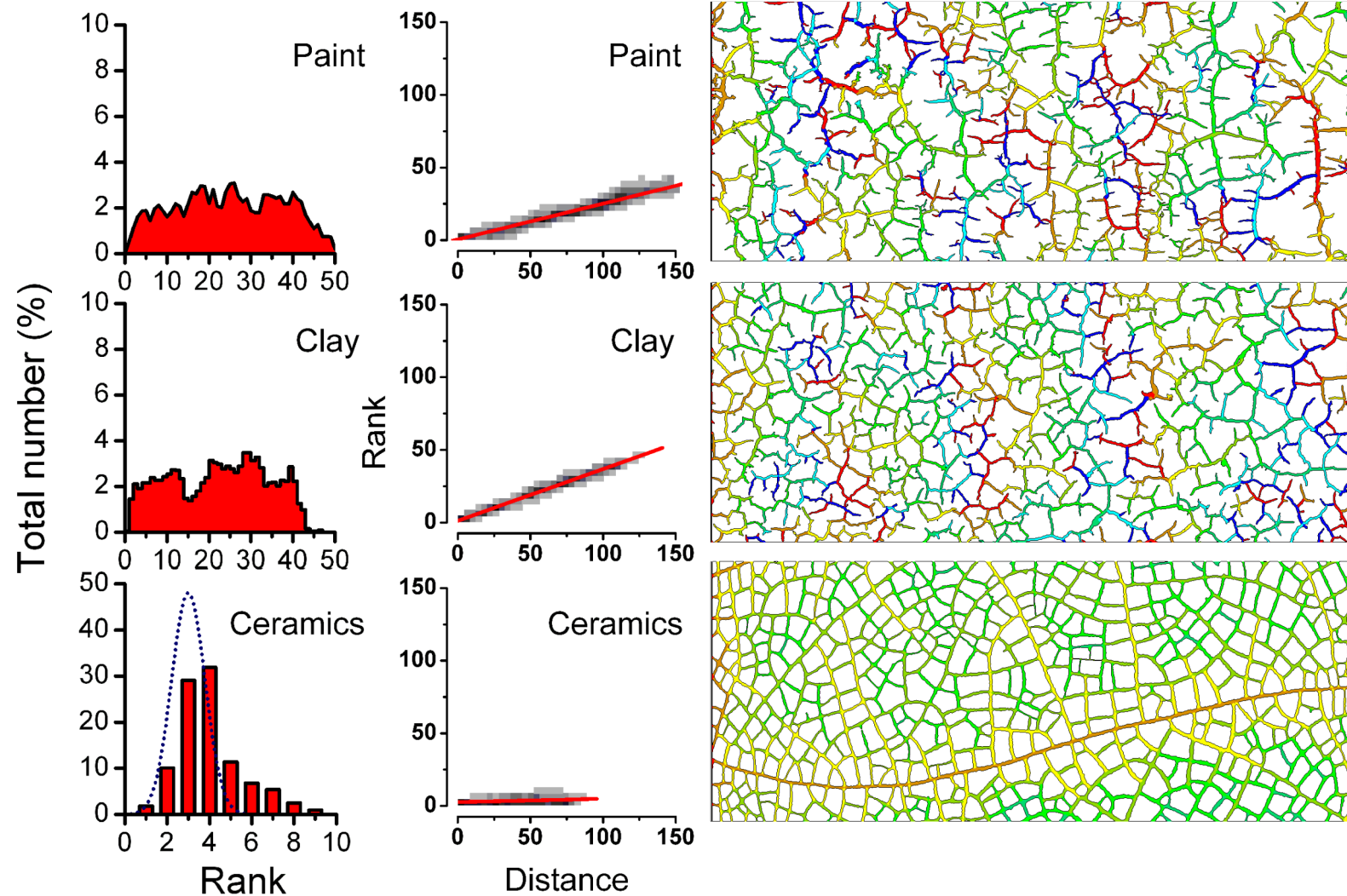
- Segment histogram
- Edge rank vs. distance



# Forces applied to the nodes

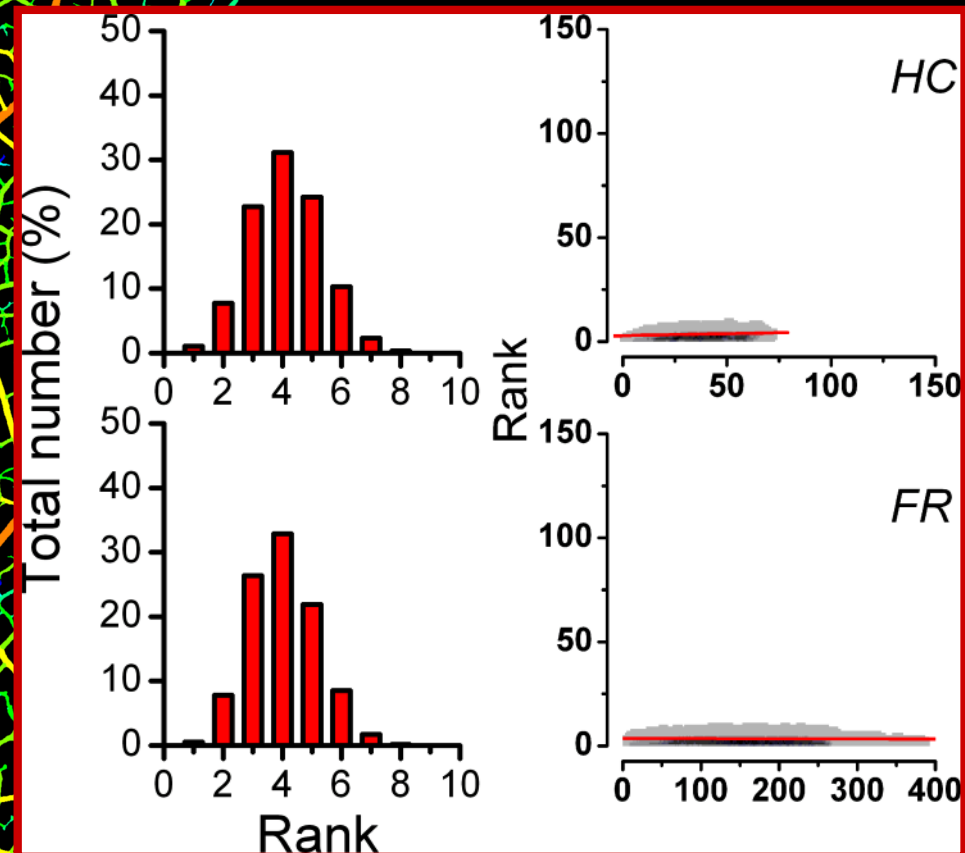


# Real systems: crack patterns

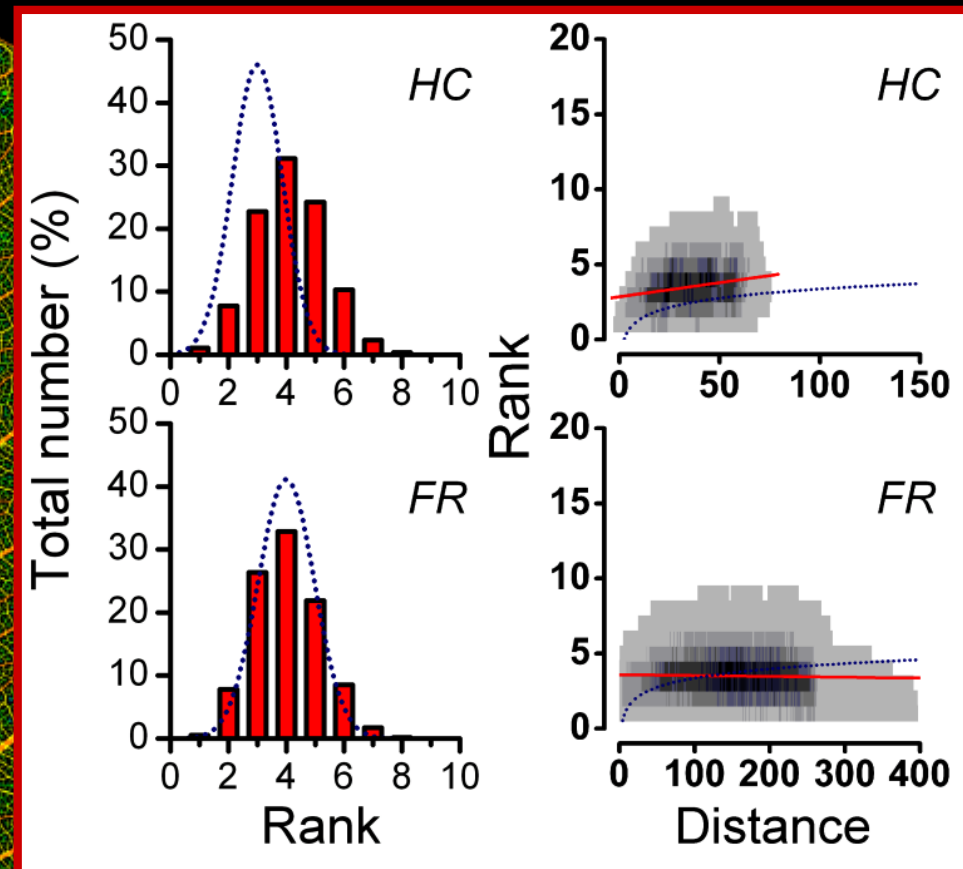




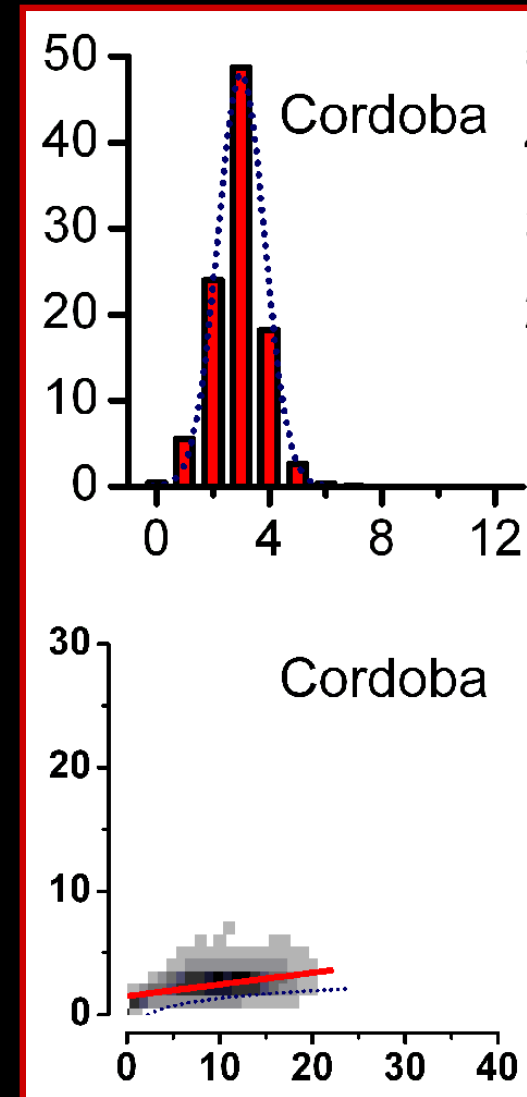
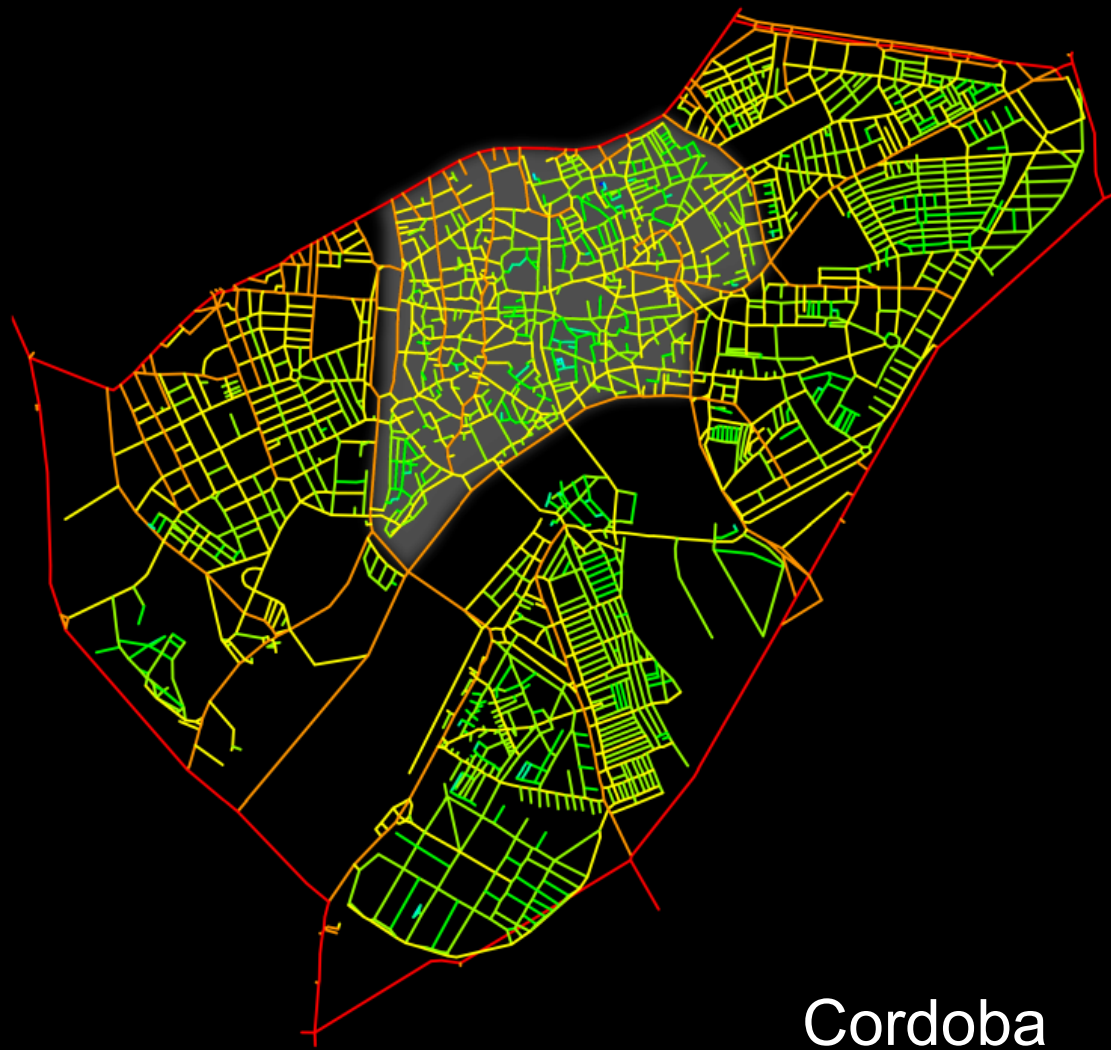
## Real systems: leaf vein networks



# Real systems: leaf vein networks

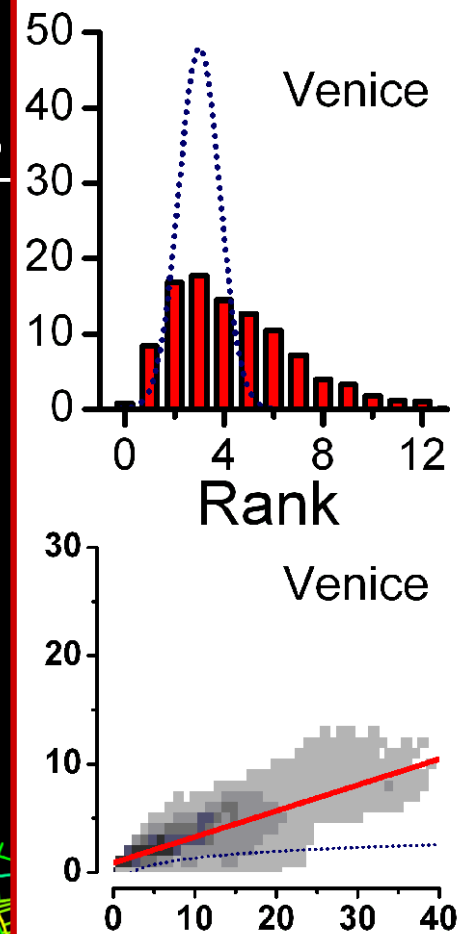
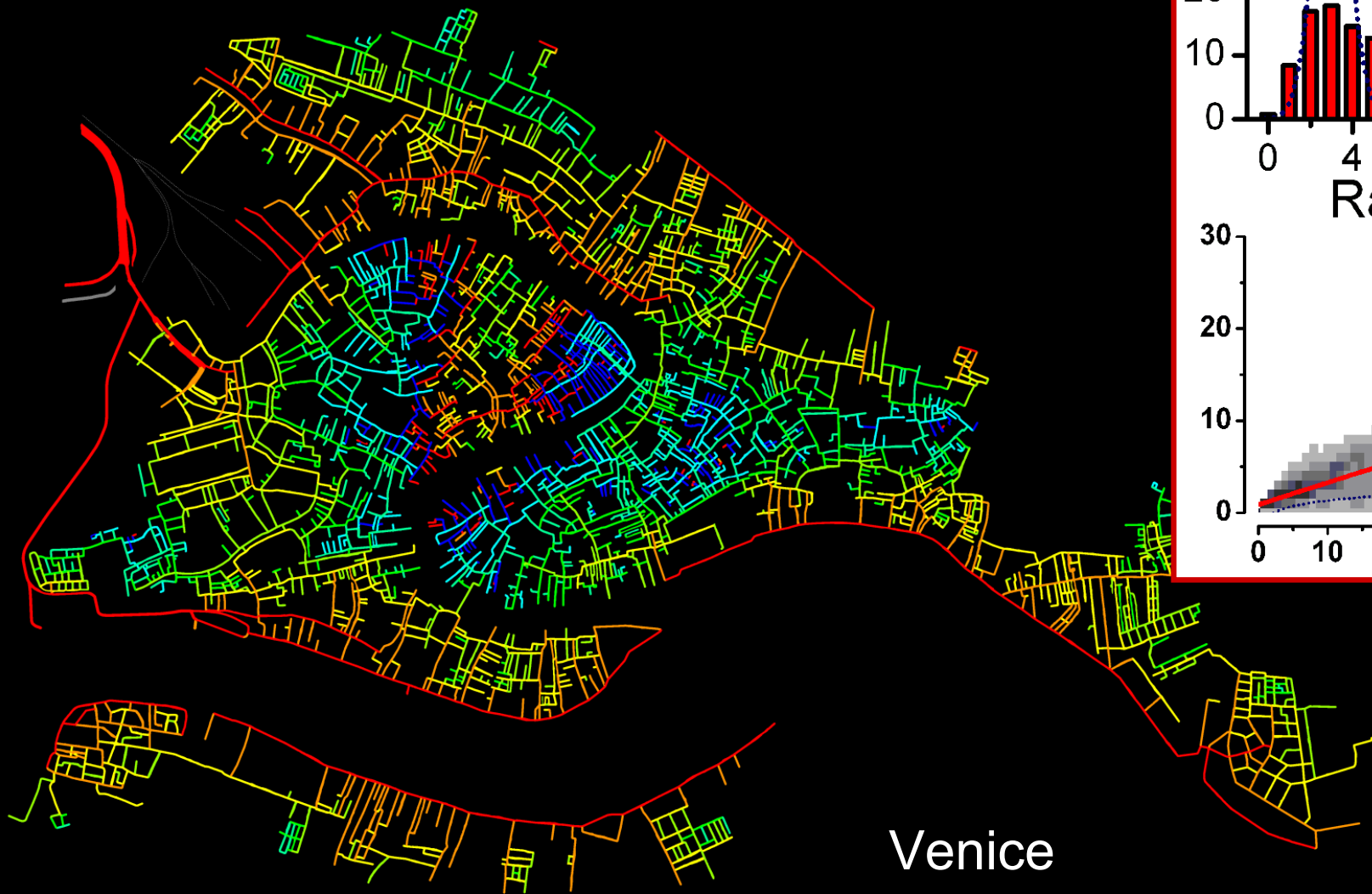


## Real systems: urban street patterns





## Real systems: urban street patterns



## **Real systems:** urban street patterns

---

Channels!



Venice







More... Map Satellite Terrain

©2006 Google - Imagery ©2006 GeoEye - Terms of Use



# Real systems: urban street patterns

---

Main entrance on  
canal







reconnection



Walking space  
= space left free...

## **Real systems:** urban street patterns

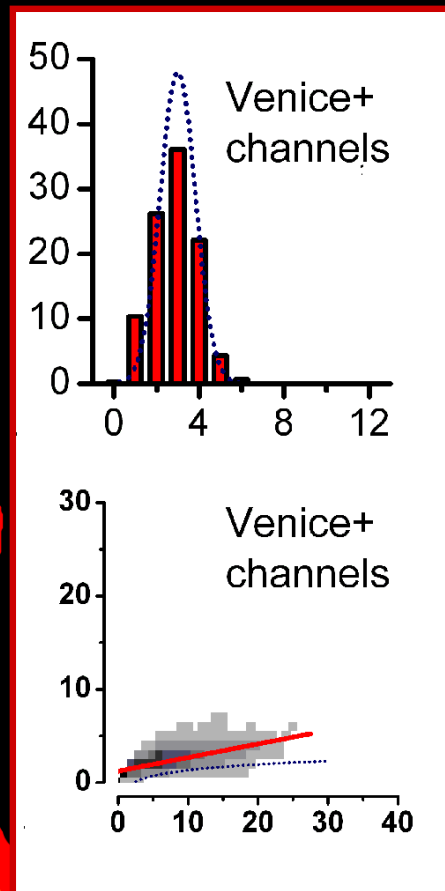
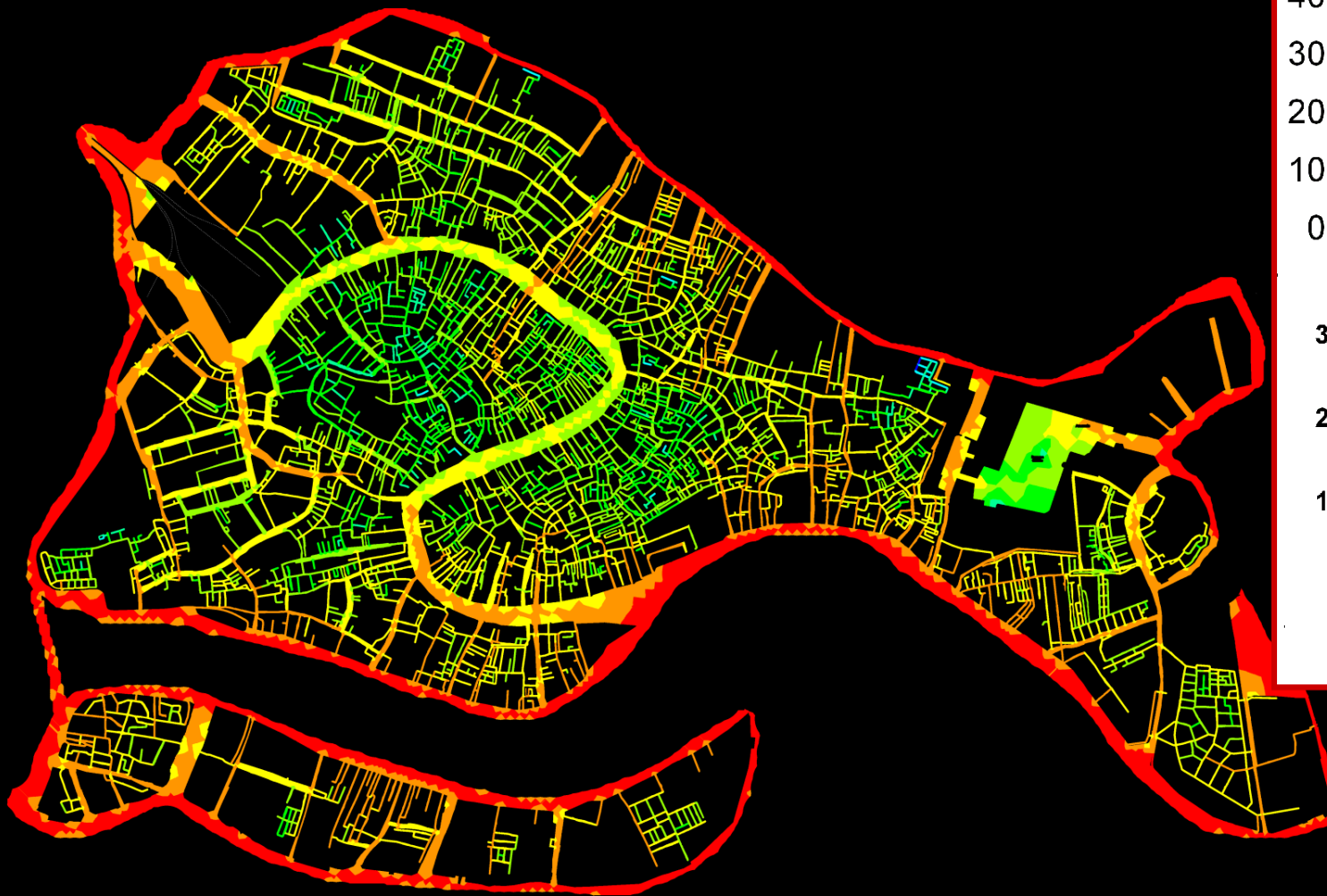
---



Channels are normal city



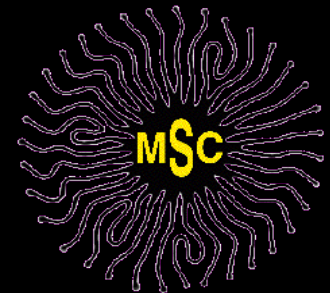
# Real systems: urban street patterns





# Three dimensional **spatial networks**

---



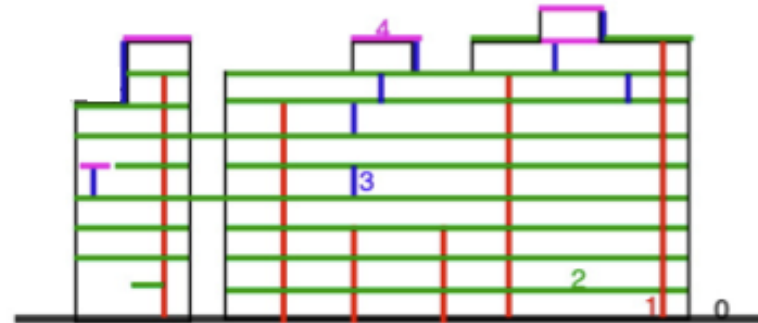
# Three dimensional spatial networks



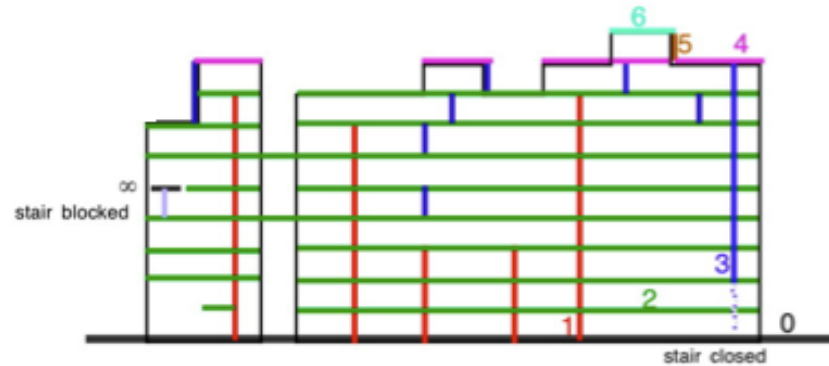
order number, (number\*(length))=(total length)

1 6,  
2 15,  
3 9,  
4 5,

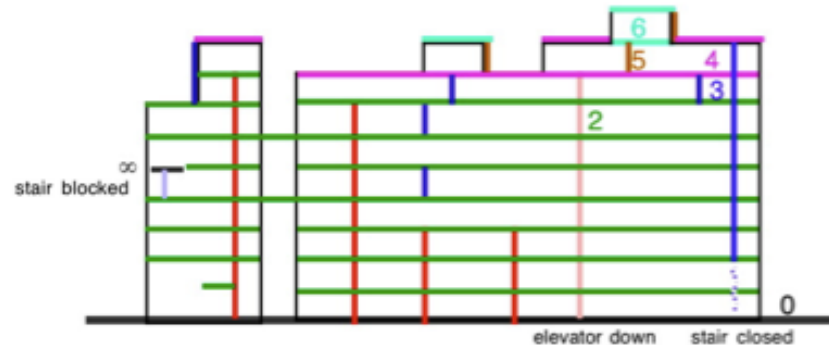
odd : stairs, elevators  
even : flats



1 5,  
2 14,  
3 8,  
4 5,  
5 1,  
6 1,  
 $\infty$  1,



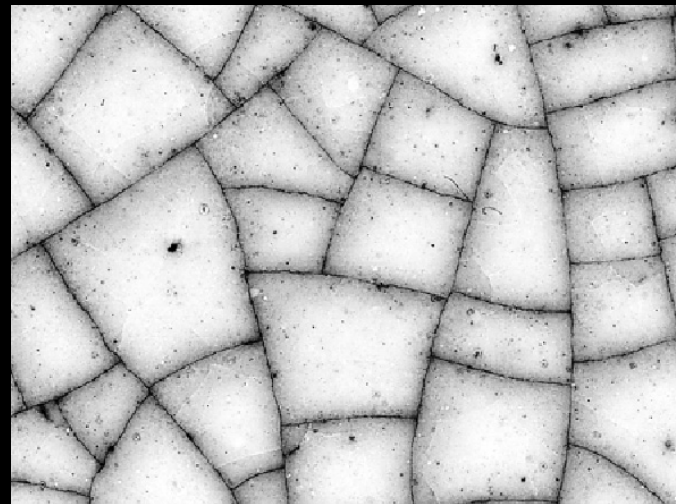
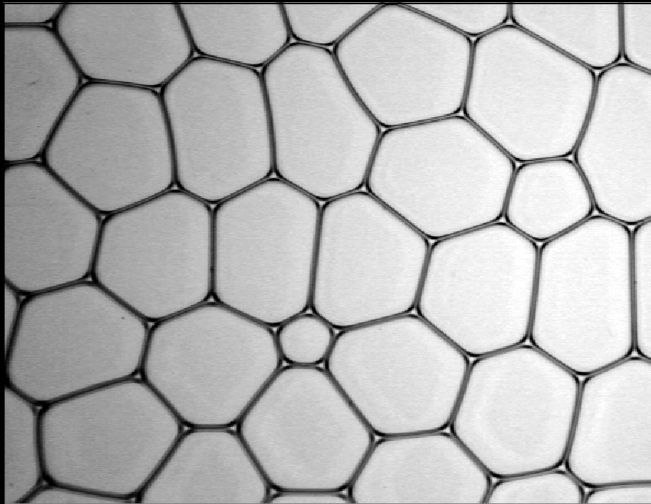
1 4,  
2 13,  
3 6,  
4 3,  
5 3,  
6 3,  
 $\infty$  1,  $\{(1)\}=\{1\}$



# Conclusions

---

1. Local geometry allows to discriminate between two classes of network-like patterns



# Conclusions

---

1. Local geometry allows to discriminate between two classes of network-like patterns
2. And to reconstruct the temporal order of appearance of different network segments

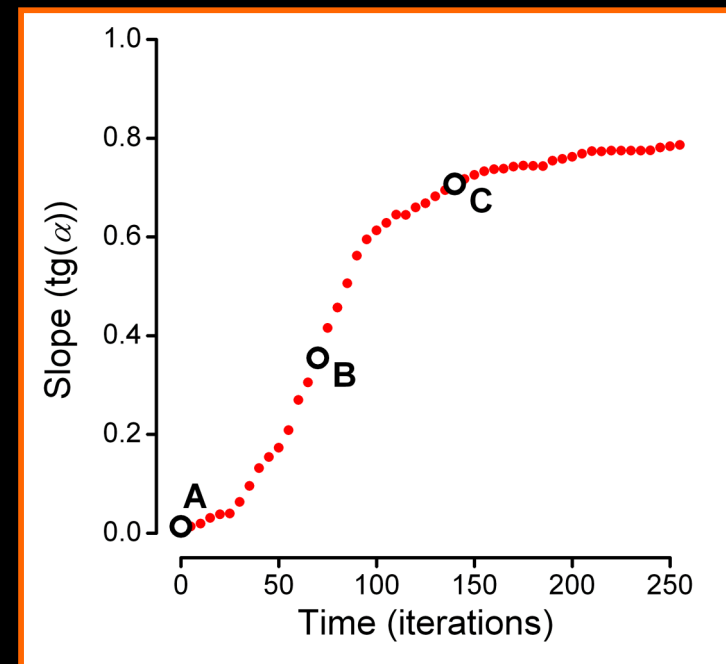




# Conclusions

---

1. Local geometry allows to discriminate between two classes of network-like patterns
2. And to reconstruct the temporal order of appearance of different network segments
3. With our algorithm we can quantify the scale over which the spatial organization persists.

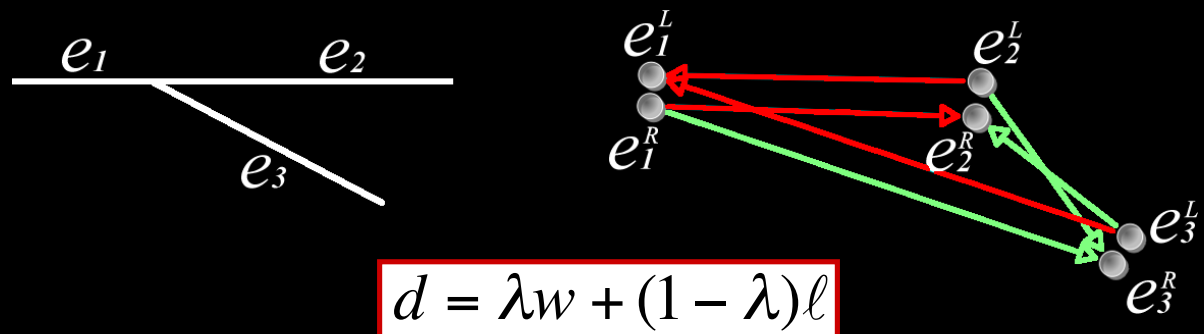




# Conclusions

---

1. Local geometry allows to discriminate between two classes of network-like patterns
2. And to reconstruct the temporal order of appearance of different network segments
3. With our algorithm we can quantify the scale over which the spatial organization persists.
4. Future work might associate (geometric) distances to the edges of the line directed graph, to explore the trade-offs between ease of orientation and short paths in different networks



# The Mesomorph project



CRCA Toulouse

Christian Jost

Jacques Gautrais

Vincent Fourcassié

Guy Theraulaz

MSC Paris

**Stéphane Douady**

LINA Nantes

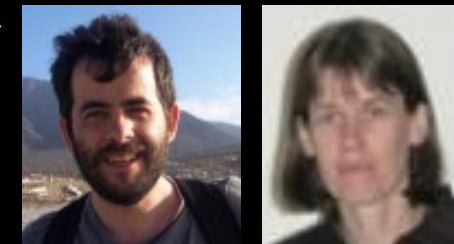
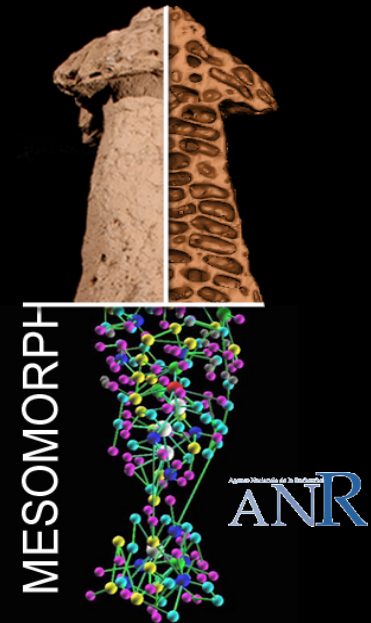
**Pascale Kuntz**

Fabien Picarougne

CSL Barcelona

Sergi Valverde

Ricard Solé



UPPSALA  
UNIVERSITET